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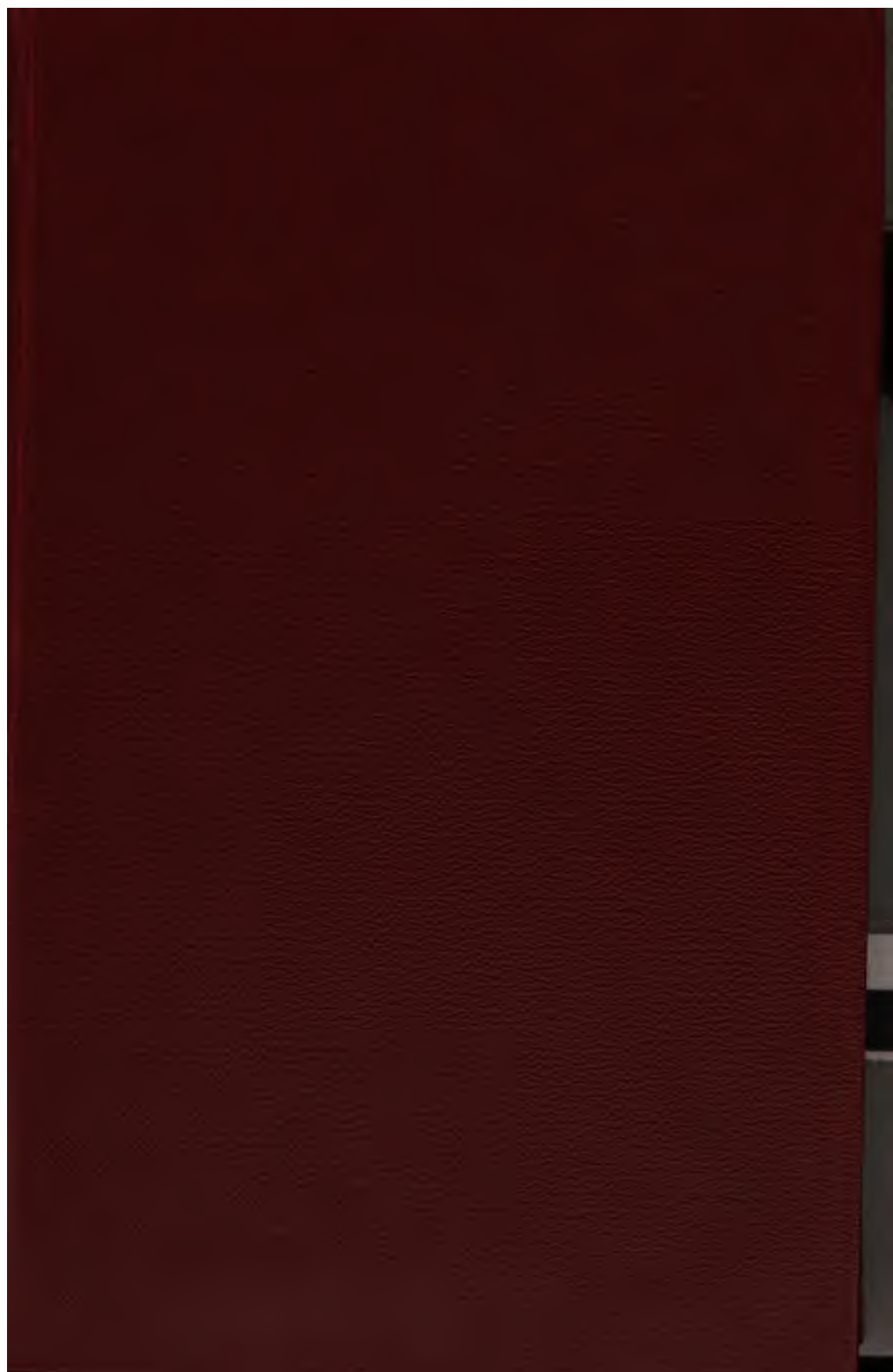
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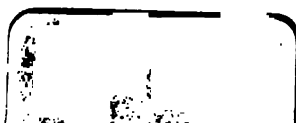


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THE  
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CONTAINING THE  
FUNDAMENTAL RULES OF THE ART,  
GEOMETRY, ARITHMETIC, & MENSURATION;  
*With the Application of those Rules to Practice.*  
THE TRUE METHOD OF  
Drawing the Ichnography and Orthography of Objects,  
GEOMETRICAL RULES FOR SHADOWS,  
ALSO THE  
FIVE ORDERS OF ARCHITECTURE;  
WITH A GREAT  
VARIETY OF BEAUTIFUL EXAMPLES,  
SELECTED FROM THE ANTIQUE;  
AND  
MANY USEFUL AND ELEGANT ORNAMENTS,  
WITH RULES FOR PROJECTING THEM.

By P. NICHOLSON, *Architect.*

Illustrated with Two Hundred and Sixteen Copper-plates, engraved in a  
superior Manner by W. Lowry, from original Drawings by the Author.

IN THREE VOLUMES.

THE SECOND EDITION WITH ADDITIONS,  
REVISED AND CORRECTED BY THE AUTHOR.

VOL. I.

London:

PRINTED FOR J. BARFIELD, WARDOUR-STREET,  
AND T. GARDINER, PRINCES-STREET CAVENDISH-SQUARE.

1809.



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## PREFACE.

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ALTHOUGH a number of publications have at different times appeared, professing to treat of the *Principles*, or *Elements of Architecture*, it is justly complained of them, that they do not fully correspond to their title. For not sufficiently entering into those mathematical principles, on which this noble art ultimately rests, and from which indeed it derives its very existence, they may rather be said to consider it *merely* as an *art*, than as a *science* also; and are more calculated to instruct the Student in drawing Architectural Plans, than to point out

and elucidate those unalterable rules, and first principles, which, however unperceived, must enter into the very essence of every plan that is correct and practicable. It is for want of attention to this circumstance, that many excellent works rather puzzle, than inform the mind of a beginner ; who ought, like the student in astronomy, to commence his inquiries by going back to the most simple elements of mathematical knowledge, there to get the real clue to his future studies, and from thence gradually and scientifically proceed to more complex problems, and more diversified plans. If the monuments of Grecian and Roman art which yet remain (remain unrivalled), it is not merely to be ascribed to their greater boldness in design, and greater expence in execution, but to that perfect knowledge and perpetual remembrance of the true principles of their art, which shines in every part

part of their edifices, uniting correctness with elegance, and permanence with grandeur. He, therefore, who wishes thoroughly to understand what the ancients have done, or to do any thing yet unattempted, must not content himself with merely drawing from their works, and then superadding the inventions of his own imagination; he must continually recur to the ground on which they trod, and make that the criterion of all his attempts,

It is principally to assist the student in this important article, that I design the following work; in which, I trust, it will appear, great pains have been taken to lay down the fundamental Principles of Architecture in a clear, distinct, and intelligible manner; and to apply the whole to practice by plain and obvious examples, illustrative of them. This I have  
endea-

endeavoured to do, so as to make the publication useful, not only to Students in Architecture, but also to Engineers, Masons, Carpenters, Carvers, Designers, Measurers, and all other persons concerned in the execution of buildings, and their several component parts. Such workmen, therefore, as aspire to any degree of superiority and taste in either of these branches, will be able from hence, by improving their leisure hours, in a short time to understand the principles of their respective occupations, and to do with taste and pleasure, what they now only do mechanically, and in servile imitation of others.

In the course of the work will be given whatever the experience of the most judicious professors has sanctioned as the best mode of effecting their professional purposes; with the reasons on which that  
pre-

preference is founded. To this will be added examples, both of Grecian and Roman antiquities; with comparisons between them, and remarks on the beauties of each. Simple, and hitherto-unpractised rules will be laid down for projecting leaves, volutes, and every other species of ornament; particular attention will be paid to the theory of shadows, both from direct and reflected light, and examples adduced of the relative degrees of light and shade on different surfaces, variously inclined to the luminary and the eye. It is no necessary part of the author's plan, to give original designs for buildings; a few, however, will be added, to exemplify the rules laid down, and to assist the student in this part of his labour.

In this Volume, the PRINCIPLES only are laid down. The GEOMETRICAL part is first attended to; and from the result

result of the theory of Geometry, a select set of problems are drawn up, many of which are entirely new, and all intimately connected with the subject in hand. They are disposed in methodical order, and are preceded by the necessary definitions.

It is not intended by this part, wholly to set aside the study of Euclid, and authors who have written on Conic Sections. An attentive perusal of their works will always amply repay the student's trouble. I have, I believe, omitted nothing material, that was connected with my design: but when the vast importance and utility of Geometry are considered, the student will never regret any pains he may take to make himself thoroughly master of every part of it.

The elegance and utility of the Ellipsis occasioning it to be introduced into  
almost

almost every species of building, I have paid particular attention to this curve; the problems relating to which, will be found particularly useful in describing elliptical and Gothic arches, finding their joints, and describing mouldings of every degree of curvature. That the reader may more perfectly understand its construction, I have shown how to draw it under various circumstances; which I have also done with regard to Conic Sections in general. The Sections of Solids are also particularly treated of; a thorough acquaintance with them being absolutely necessary for understanding the theory and disposition of Shadows: in explaining which, I flatter myself this work will be found to exceed every other hitherto published.

Number, as well as magnitude, being concerned in Architecture, ARITHME-

TIC follows next. The importance of this in forming estimates, both of materials and expence, in giving rules for measuring, and fixing a price on work, &c. is sufficiently obvious. Here I have endeavoured to be as concise and clear as the nature of the subject will admit. All operations purely arithmetical, being either an application singly of the four primary rules (viz. Addition, Subtraction, Multiplication, and Division), or else compounded of them, care has been taken to define the terms clearly, and to give the proper axioms under their respective heads.

In stating questions in Proportion, whether simple, inverse, or compound, I have shown a more general and easy, as well as a more rational method than has hitherto been made known, by taking together all the component parts of the given

given cause for the first term, and the component parts of the given effect for the second; then taking the component parts of any other cause producing similar effects for the third term, and the effect performed by these for the fourth; all operations in proportion will be reduced to the form of four terms; and if all the terms are complete, the product of the two extremes will always be equal to the product of the two means.

The best method of computing vulgar, decimal, and duodecimal fractions, and of extracting the square and cube roots, in order to facilitate the knowledge of Mensuration, concludes this part.

MENSURATION itself is then explained. This, showing the proportion one magnitude bears to another of the same kind, is necessary to enable the

architect to proportion the scantlings of his timber, and to give strength and stability to his design. I have therefore given rules for measuring all the most useful kinds of figures, in the shortest way, as applied to lines, superficies, and solids. The customary methods of measuring the several Artificers Works, are then pointed out in a plain and familiar manner, with examples at full length, for the assistance of workmen in general, that they may be enabled to detect the impositions of fraudulent men, who too often find their way into every profession, and the mistakes that others may inadvertently make. The method of measuring groins upon any rectangular plan, is afterwards subjoined.

P. NICHOLSON.

*October 6, 1808.*

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*End of the Contents of the First Volume.*



# **PRACTICAL GEOMETRY;**

CONTAINING

A NUMBER OF SELECT

**Problems,**

MANY OF WHICH ARE ENTIRELY NEW:

Necessary to be understood by all who intend to become

PROFICIENTS

IN THE

PRINCIPLES OF ARCHITECTURE.





## PRACTICAL GEOMETRY.

### DEFINITIONS.

1. **GEOMETRY** is that science which treats of the descriptions and properties of magnitudes in general.

2. A *point* has neither parts nor magnitude.

3. A *line* is length, without breadth or thickness.

4. A *superfices* has length and breadth only.

5. A *solid* is a figure of three dimensions, having length, breadth, and thickness.—Hence surfaces are the extremities of solids, and lines the extremities of surfaces, and points the extremities of lines.

6. *Lines* are either right, curved, or mixed of these two.

7. A *right* or *straight line* lies in the same direction between its extremities, and is the shortest distance between two points.

8. A *curve* continually changes its direction between its extreme points.

9. *Lines* are either parallel, oblique, perpendicular, or tangential.

10. *Parallel lines* are always at the same distance, and will never meet though ever so far produced.

11. *Oblique right lines* in the same plane, change their distance, and would meet, if produced, as A.

12. One *line* is perpendicular to another when it inclines no more to one side than another, as B.

13. One *line* is a tangent to another when it touches it without cutting, when both are produced, as C.

14. An *angle* is the inclination of two lines towards one another, meeting in a point, as D.

15. *Angles* are either right, acute, or oblique, as E.

16. A *right angle* is that which is made by one line perpendicular to another, or when the angles on each side are equal, as F.

17. An *acute angle* is less than a right angle, as G.

18. An *obtuse angle* is greater than a right angle, as H.

19. A *superfices* is either plane or curved.

20. A *plane*, or *plane surface*, is that to which a right line will every way coincide; but if not, it is curved.

21. *Plane figures* are bounded either by right lines, or curves.

22. *Plane figures*, bounded by right lines, have names according to the number of their sides, or angles, for they have as many sides as angles—the least number is three.

23. An *equilateral triangle* is that whose three sides are equal, as I.

24. An *isosceles triangle* has only two sides equal, as K.

25. A *scalene triangle* has all sides unequal, as L.

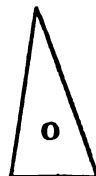
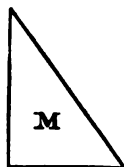
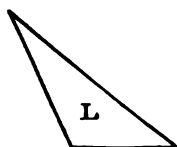
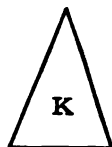
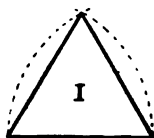
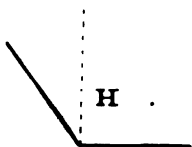
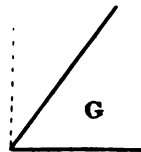
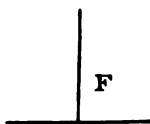
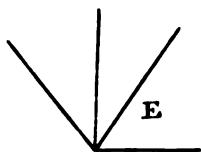
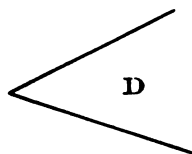
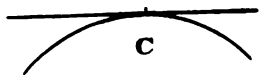
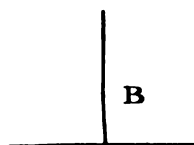
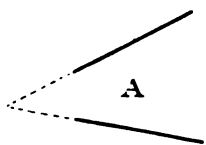
26. A *right angled triangle* has one right angle, as M.

27. Other *triangles* are oblique angled, and are either obtuse or acute.

28. An *acute angled triangle* has all its angles acute, as N or O.

29. An

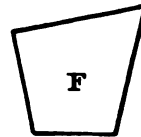
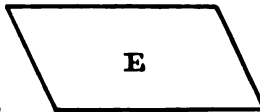
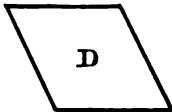
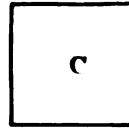
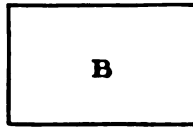
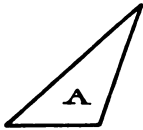
*Pl. 1.*







*Pl. 2.*



29. An *obtuse angled triangle* has one obtuse angle, as A.
30. A *figure* of four sides and angles, is called a *quadrangle*, or *quadrilateral*, as B, C, D, E, F, and G.
31. A *parallelogram*, is a quadrilateral, which has both pairs of its opposite sides parallel, as B, C, D, and E; and takes the following particular names.
32. A *rectangle*, is a parallelogram, having all its angles right ones, as B and C.
33. A *square*, is an equilateral rectangle, having all its sides equal, and all its angles right ones, as C.
34. A *rhombus*, is an equilateral parallelogram, whose angles are oblique, as D.
35. A *rhomboid*, is an oblique angled parallelogram, as E.
36. A *trapezium*, is a quadrilateral, which has neither pair of its sides parallel, as F.
37. A *trapezoid* hath only one pair of its opposite sides parallel, as G.
38. *Plane figures* having more than four sides, are in general called *polygons*, and receive other particular names, according to the number of their sides or angles.
39. A *pentagon* is a polygon of five sides, a *hexagon* hath six sides, a *heptagon* seven, an *octagon* eight, a *nonagon* nine, a *decagon* ten, an *undecagon* eleven, and a *dodecagon* twelve sides.
40. A *regular polygon* hath all its sides and angles equal; and if they are not equal, the polygon is *irregular*.
41. An *equilateral triangle* is also a regular figure of three sides, and a *square* is one of four; the former being called a *trigon*, and the latter a *tetragon*.
42. A *circle* is a plane figure, bounded by a curve line, called the *circumference*, which is every-where equidistant from a certain point within called its *centre*.

43. The *radius* of a circle is a right line drawn from the centre to the circumference, as  $a b$ , at A.

44. A *diameter* of a circle is a right line drawn through the centre, terminating on both sides of the circumference, as  $c d$ , at B.

45. An *arc* of a circle is any part of the circumference.

46. A *chord* is a right line joining the extremities of an arc, as  $a b$ , at C.

47. A *segment* is any part of a circle, bounded by an arc and its chord, as D.

48. A *semicircle* is half the circle, or a segment cut off by the diameter, as E.

49. A *sector* is any part of a circle bounded by an arc and two radii, drawn to its extremities, as F.

50. A *quadrant*, or quarter of a circle, is a sector, having a quarter of the circumference for its arc, and the two radii are perpendicular to each other, as G.

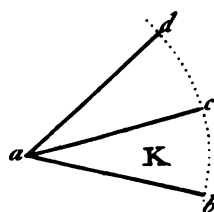
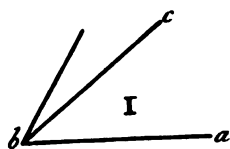
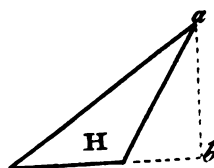
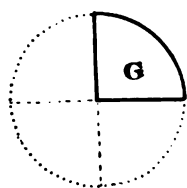
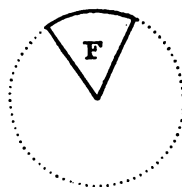
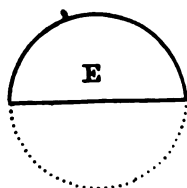
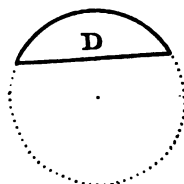
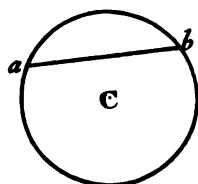
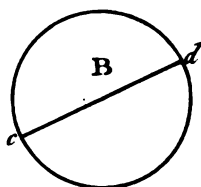
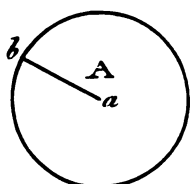
51. The height or altitude of any figure, is a perpendicular let fall from an angle, or its vertex, to the opposite side, called the *base*, as  $a b$ , at H.

52. When an angle is denoted by three letters, the middle one is the place of the angle, and the other two denote the sides containing that angle; thus, let  $a b c$  be the angle as I, then  $b$  will be the angular point, and  $a b$ , and  $b c$ , will be the two sides containing that angle.

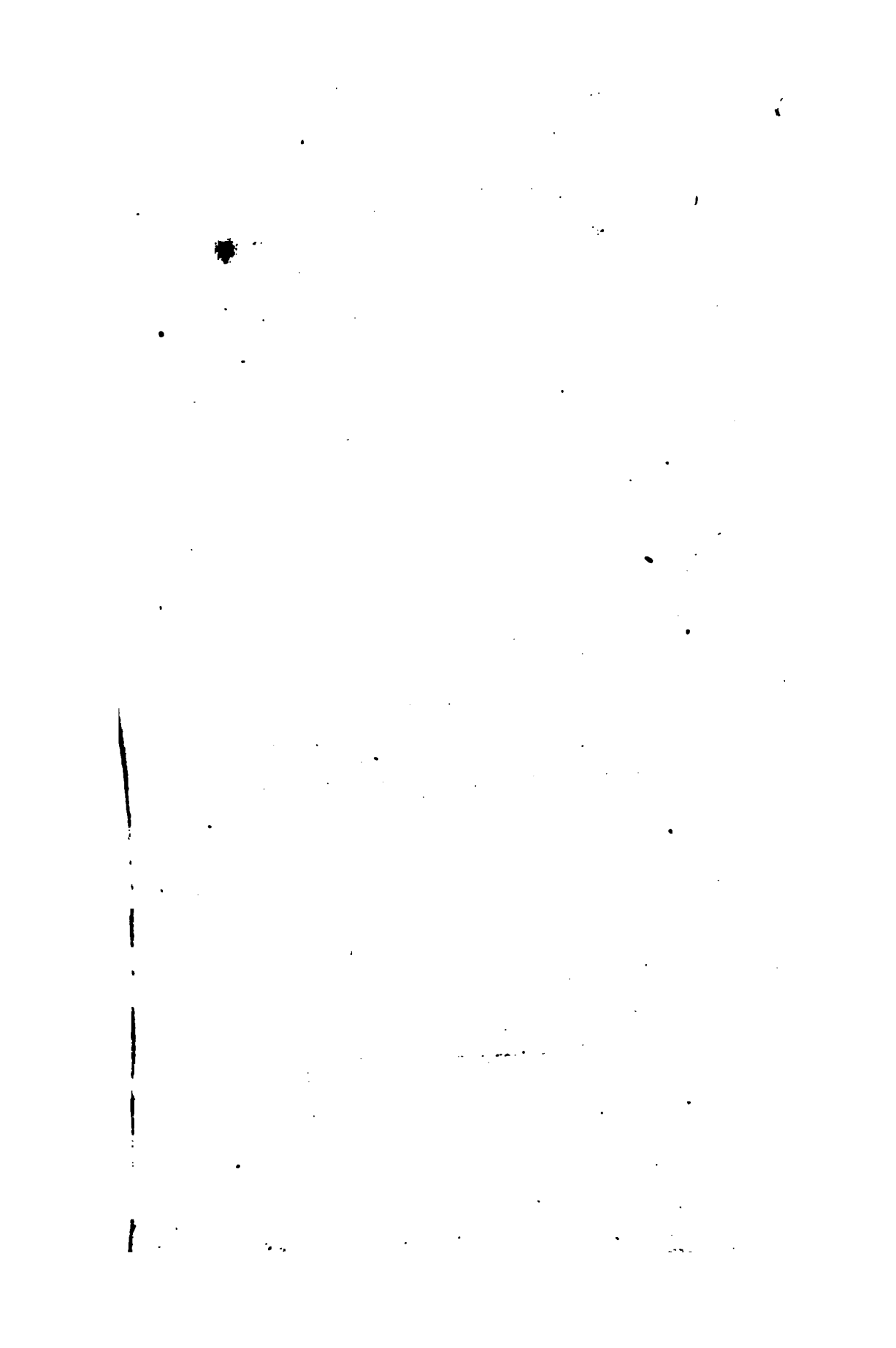
53. The measure of any right lined angle, is an arc of any circle contained between the two lines which form the angle, the angular point being in the centre, as K. Thus if the arc  $b c d$  be double of the arc  $b c$  then the angle  $b a d$ , will be double that of  $b a c$ .

## PROBLEM

*Pl. 3.*

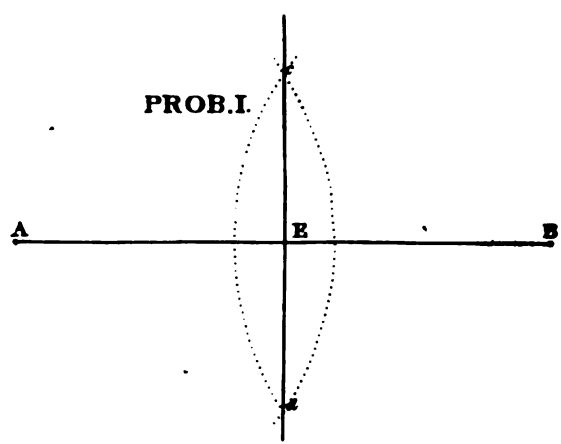






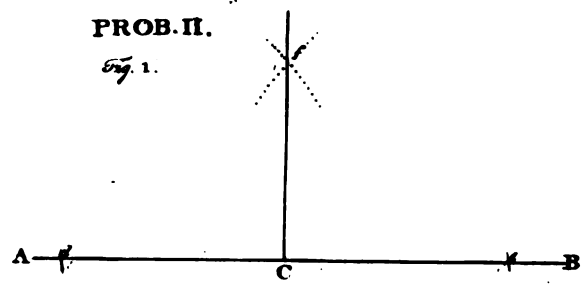
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PROB. I.

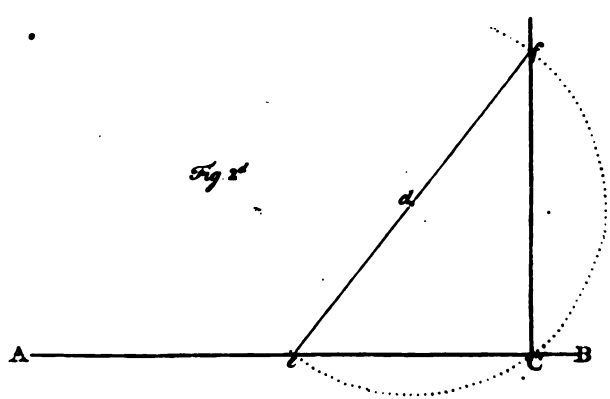


PROB. II.

*Fig. 1.*



*Fig. 2.*



## P R O B L E M S.

### PROBLEM I.

To bisect a given line A B.

1. From the points A and B, as centres, with any distance greater than half A B, describe arcs cutting each other in *c* and *d*.
2. Draw the line *c d* and the point E, where it cuts A B, will be the middle of the line required,

### PROBLEM II.

From a given point C, in a given right line A B,  
to erect a perpendicular.

FIGURE 1. *When the point is near the middle of the line,*

1. On each side of the point C take any two equal distances C *d* and C *e*.
2. From *d* and *e*, with any radius greater than C *d*, or C *e*, describe two arcs cutting each other in *f*.
3. Through the points *f* C, draw the line *f* C, and it will be the perpendicular required,

FIG. II. *When the point is at, or near, the end of the line.*

1. Take any point *d* above the line, and with the radius or distance *d* C, describe the arc *e* C *f*, cutting A B in *e* and C.
2. Through the centre *d* and the point *e*, draw the line *e d f*, cutting the arc *e* C *f*, in *f*.
3. Through the points *f* C, draw the line *f* C, and it will be the perpendicular required.

PROBLEM

## PROBLEM III.

From a given point  $C$ , out of a given right line  $A B$ , to let fall a perpendicular

1. From the point  $C$ , with any radius, describe the arc  $d e$ , cutting  $A B$  in  $e$  and  $d$ .
2. From the points  $e d$  with the same, or any other radius, describe two arcs cutting each other, in  $f$ .
3. Through the points  $C f$ , draw the line  $C D f$ , and  $C D$  will be the perpendicular required.

## PROBLEM IV.

At a given point  $D$ , upon the right line  $D E$ , to make an angle equal to a given angle  $a B b$ .

1. From the point  $B$ , with any radius, describe the arc  $a b$ , cutting the legs  $B a$ ,  $B b$ , in the points  $a$  and  $b$ .
2. Draw the line  $D e$ , and from the point  $D$ , with the same radius as before, describe the arc  $e f$ , cutting  $D E$  in  $e$ .
3. Take the distance  $b a$ , and apply it to the arc  $e f$ , from  $e$  to  $f$ .
4. Through the points  $D f$ , draw the line  $D f$ , and the angle  $e D f$ , will be equal to the angle  $b B a$ , as was required.

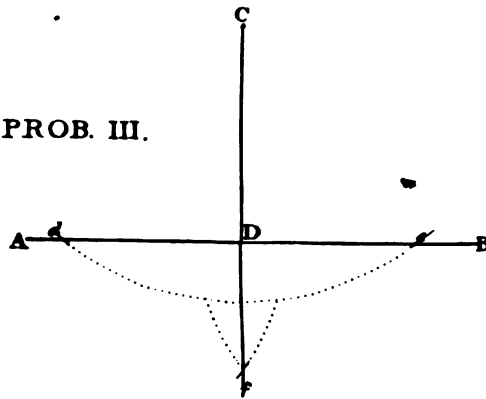
## PROBLEM V.

To divide a given angle  $A B C$  into two equal angles.

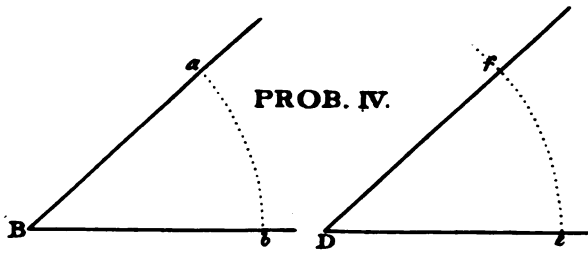
1. From the point  $B$ , with any radius, describe the arc  $A C$ .
2. From  $A$  and  $C$ , with the same or any other radius, describe arcs cutting each other in  $d$ .
3. Draw the line  $B d$ , and it will bisect the angle  $A B C$ , as was required.

PROBLEM

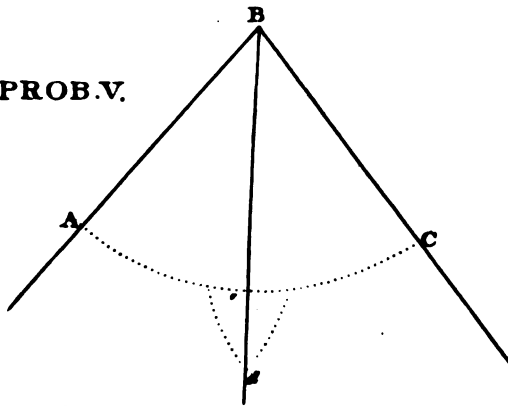
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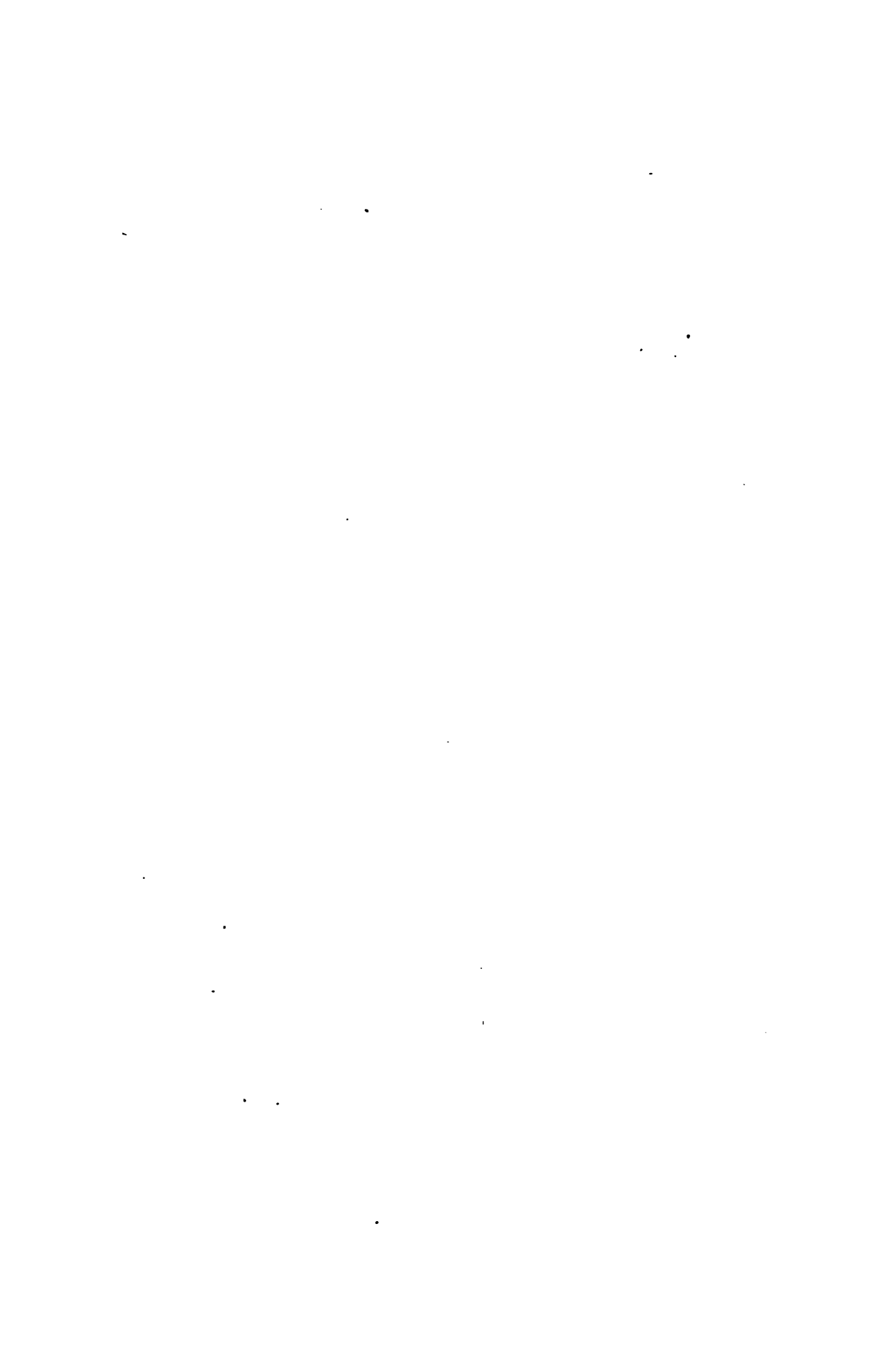


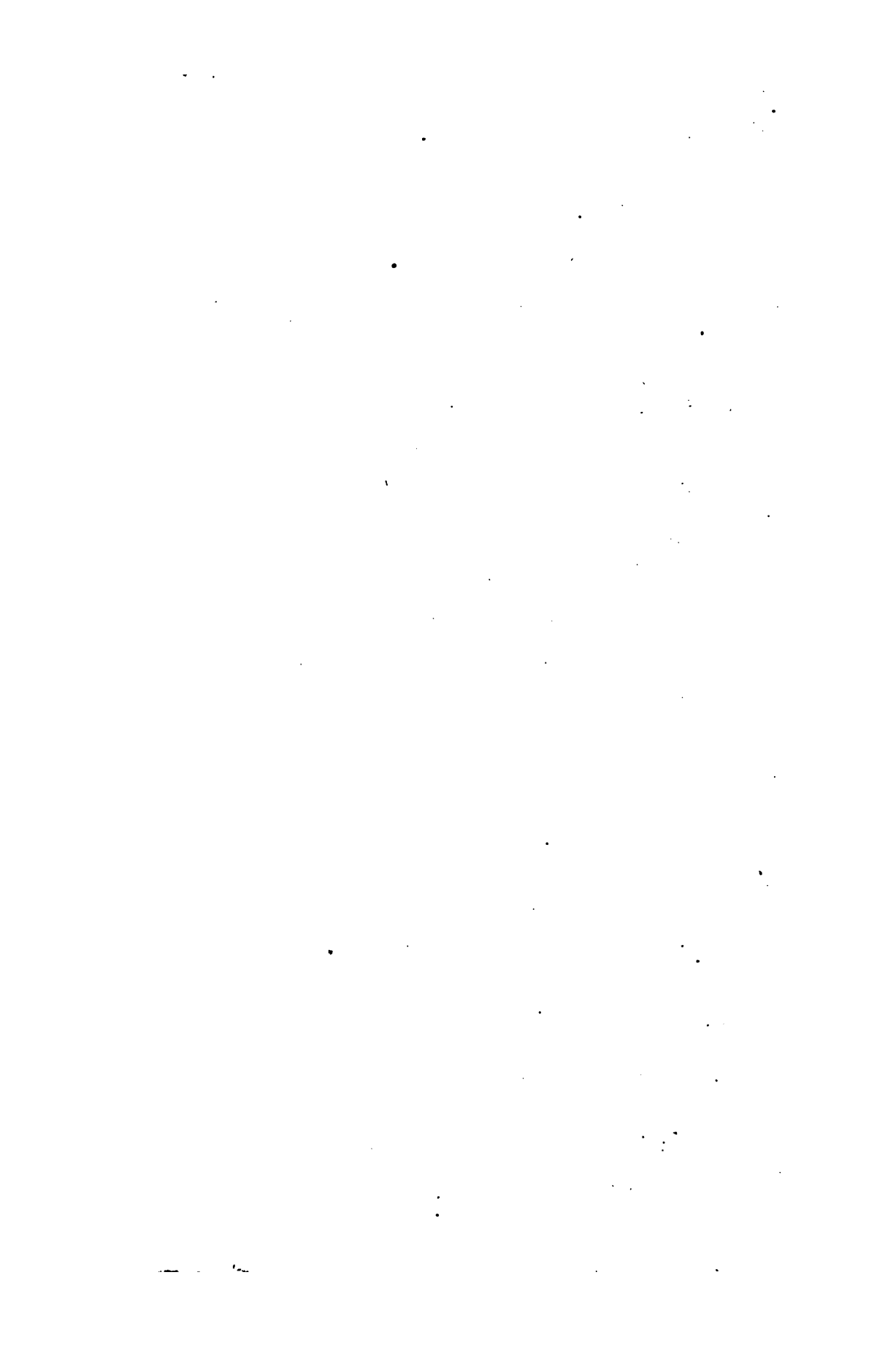
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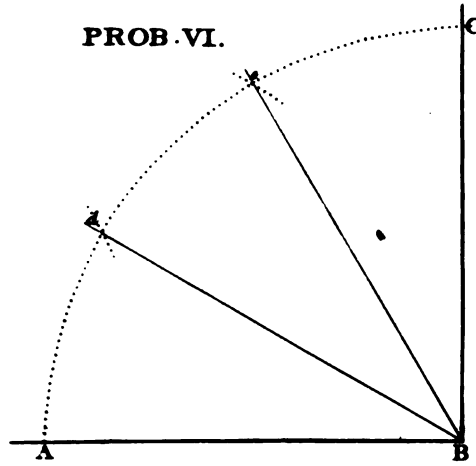
PROB. V.







PROB. VI.



PROB. VII.

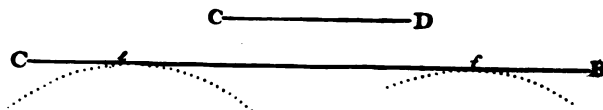
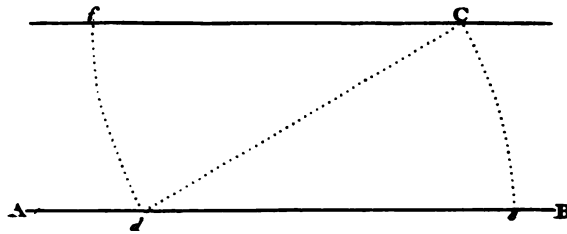


Fig. 2.



## PROBLEM VI.

To trisect or divide a right angle  $A B C$  into three equal angles.

1. From the point  $B$ , with any radius  $B A$ , describe the arc  $A C$ , cutting the legs  $B A$ , and  $B C$ , in  $A$  and  $C$ .
2. From the point  $A$ , and  $C$ , with the radius  $A B$ , or  $B C$ , cross the arc  $A C$  in  $d$ , and  $e$ .
3. Through the points  $e d$ , draw the lines  $B e$ ,  $B d$ , and they will trisect the angle as was required.

## PROBLEM VII.

Through a given point  $C$ , to draw a line parallel to a given line  $A B$ .

1. Take any point  $d$ , in  $A B$ , upon  $d$  and  $C$ , with the distance  $C d$ , describe two arcs  $e C$ , and  $d f$ , cutting the line  $A B$ , in  $e$  and  $d$ .
2. Make  $d f$  equal to  $e C$ ; through  $C$  and  $f$ , draw  $C f$ , it will be the line required.

FIG. II. *When the parallel is to be at a given distance  $C D$  from  $A B$ .*

1. From any two points  $e$  and  $d$ , in the line  $A B$ , with a radius equal to  $C D$ , describe the arcs  $e$  and  $f$ .
2. Draw the line  $C B$ , to touch those arcs without cutting them, and it will be parallel to  $A B$  as was required.

PROBLEM

## PROBLEM VIII.

To divide a given line  $A B$  into any proposed number of equal parts.

1. From  $A$ , one end of the line, draw  $A c$ , making any angle, with  $A B$ ; and from  $B$ , the other end, draw  $B d$ , making the angle  $A B d$  equal to  $B A c$ .
  2. In each of the lines  $A c$ ,  $B d$ , beginning at  $A$  and  $B$ , set off as many equal parts, of any length, as  $A B$  is to be divided into.
  3. Join the points  $A 5$ ,  $1 4$ ,  $2 3$ , &c., and  $A B$  will be divided as was required.
- 

## PROBLEM IX.

To find the centre of a given circle, or one already described.

- 1 Draw any chord  $A B$ , and bisect it with the perpendicular  $C D$
  2. Bisect  $C D$  with the diameter  $E f$ , and the intersection  $O$  will be the centre required.
- 

## PROBLEM X.

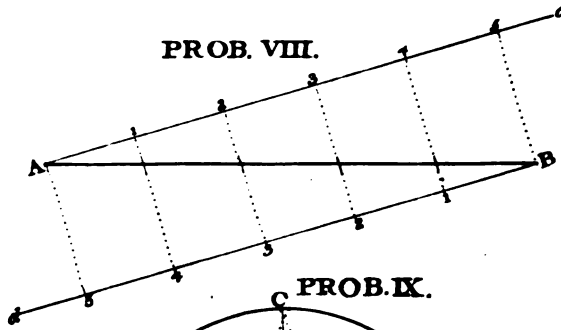
To draw a tangent to a given circle, that shall pass through a given point  $A$ .

1. From the centre  $O$ , draw the radius  $O A$ .
2. Through the point  $A$ , draw  $D E$  perpendicular to  $O A$ , and it will be the tangent required.

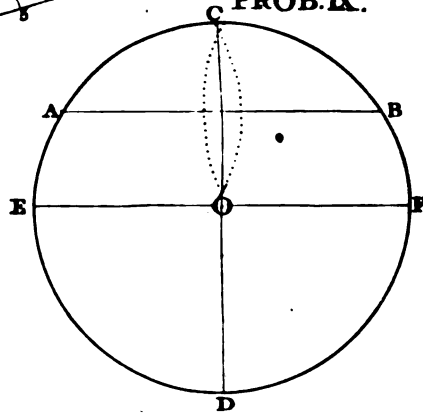
PROBLEM

*Plate 7.*

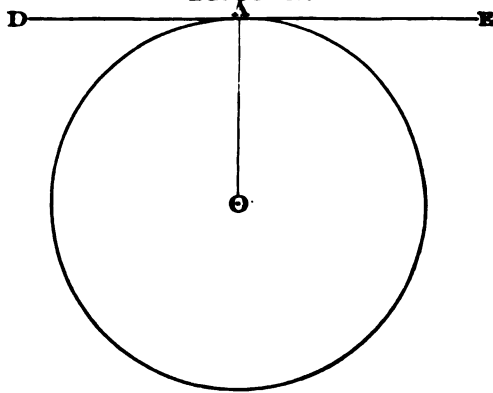
**PROB. VIII.**



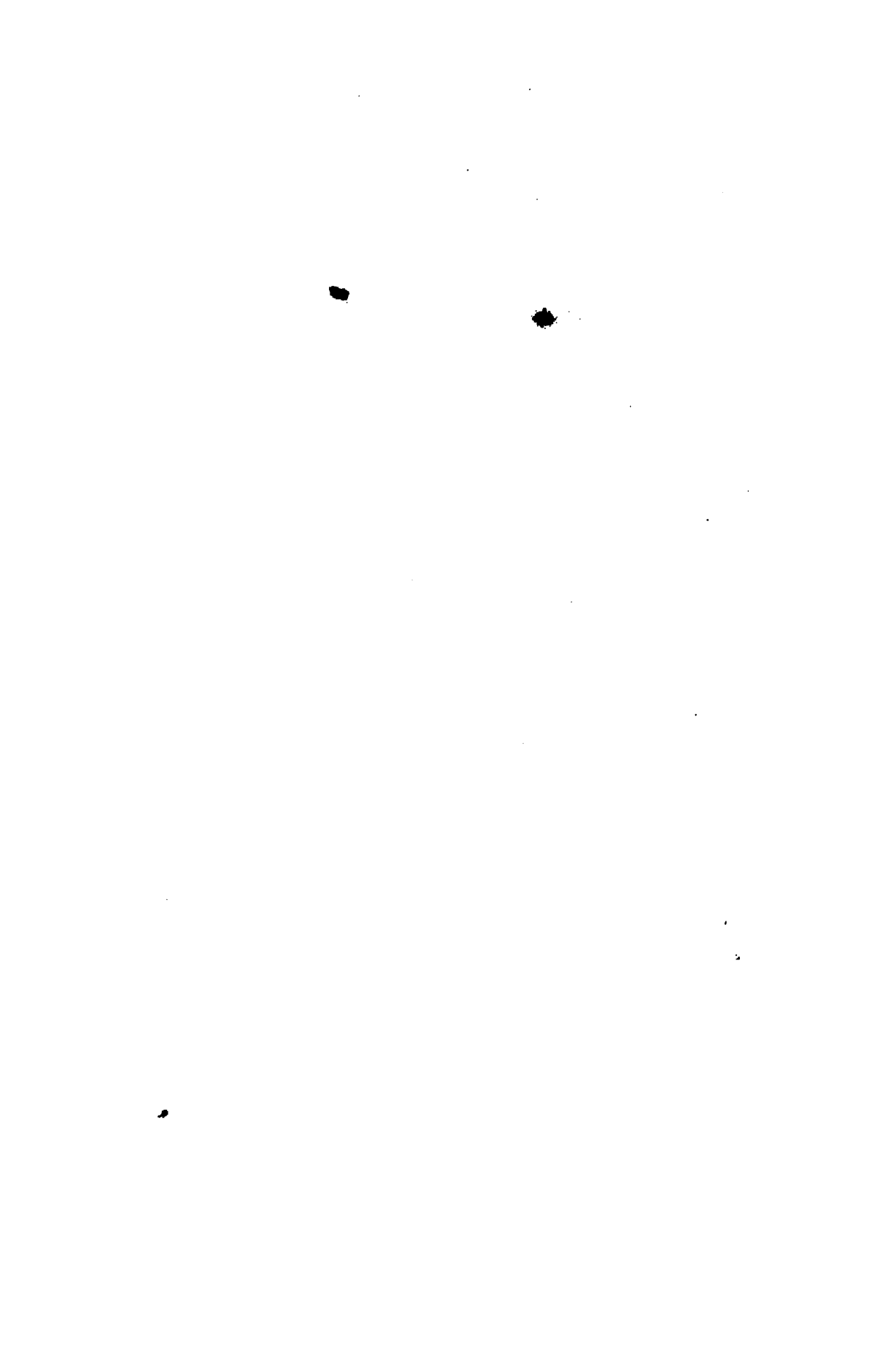
**PROB. IX.**

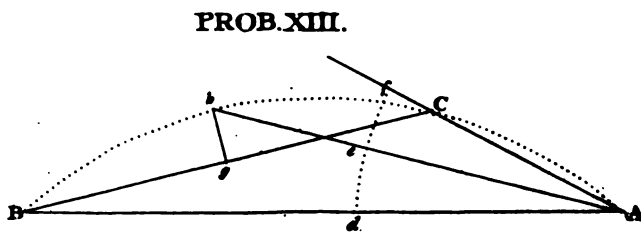
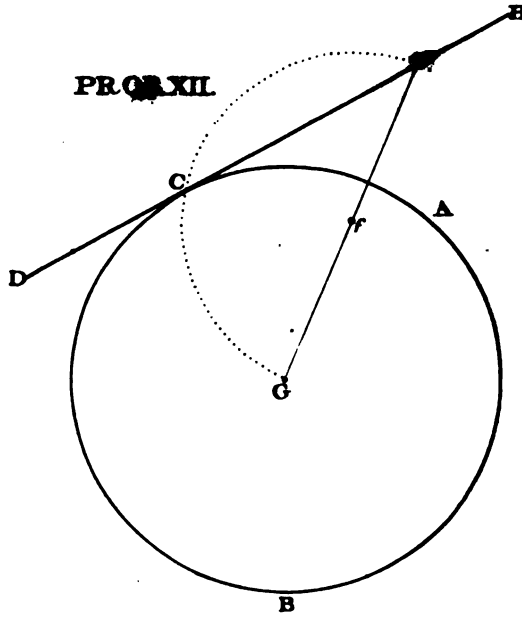
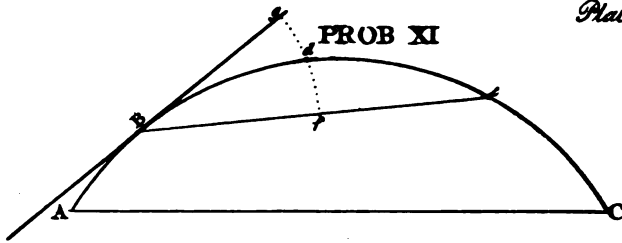


**PROB. X.**









## PROBLEM XI.

To draw a tangent to a circle, or any segment of a circle  $A B C$ , through a given point  $B$ , without making use of the centre of the circle.

1. Take any two equal divisions upon the circle, from the given point  $B$ , towards  $d$  and  $e$ , draw the chord  $e B$ .
2. Upon  $B$ , as a centre, with the distance  $B d$ , describe the arc,  $f d g$ , cutting the chord  $e B$  in  $f$ .
3. Make  $d g$  equal to  $d f$ , through  $g$  draw  $g B$ , and it will be the tangent required.

## PROBLEM XII.

A circle  $A B C$  being given, and a tangent  $D H$  to that circle, to find the point contact.

1. Take any point  $e$ , in the tangent  $D H$ ; from  $e$ , to the centre of the circle  $G$ , draw  $e G$ .
2. Bisect  $e G$  in  $f$ , and with the radius  $f e$ , or  $f G$ , describe the semicircle  $e C G$ , cutting the tangent and the circle in  $C$ , it will be the point required.

## PROBLEM XIII.

Given three points,  $A, B, C$ , not in a straight line, to find a number of points, lying between them, so that they shall all be in the circumference of a circle, without drawing any part of the circle, or finding the centre.

1. From  $A$ , through  $B$  and  $C$ , draw  $A B$  and  $A C$ .
2. On  $A$ , as a centre, with any radius  $A f$ , describe an arc  $f e d$ , cutting  $A B$  in  $d$ , and  $A C$  in  $f$ .
3. Bisect the arc  $d f$  in  $e$ , through  $e$ , draw  $A e h$ .
4. Join  $C B$ , bisect it in  $g$ , draw  $g h$  perpendicular, cutting  $A e h$  at  $h$ , then  $h$  will also be in the same circumference with  $A, C, B$ . In the same manner may a point be found between  $C h$ , and  $h B$ .

## PROBLEM XIV.

Given three points A, B, C, not in a right line, to find another point without these points, so that the four points shall all be in the circumference of a circle, without drawing any part of the circle, or finding the centre.

1. Draw A *e* and A C, from A, through B and C.
2. On A, as a centre with any radius A *e*, draw the arc *e f g*, cutting A B in *e*, and A C in *f*.
3. Make *f g* equal to *f e*, through A and *g* draw A *g* indefinitely towards *d*.
4. Upon C, with the distance C B, cross the line A *g* at *d*, it will be the point required.

If a fifth point, or any other number of points are required, the process will be the same.

## PROBLEM XV.

Given three points A, B, C, not in a straight line, to draw a circle through them.

1. Bisect the lines A B, and B C, by the perpendiculars, meeting at *d*.
2. Upon *d*, with the distance *d A*, *d B*, or *d C*, describe A B C, it will be the circle required.

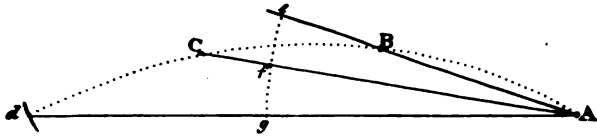
## PROBLEM XVI.

To describe the segment of a circle to any length A B, and breadth C D.

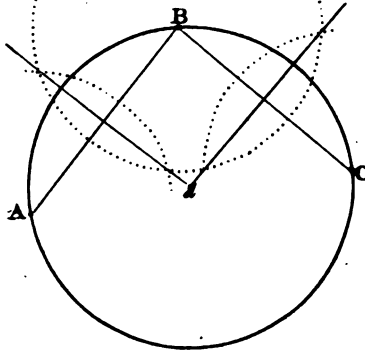
1. Bisect A B, by the perpendicular D *g*, cutting A B, in C.
2. From *c*, make *c D* on the perpendicular, equal to C D.
3. Bisect A D, by a perpendicular *e f*, cutting D *g*, in *g*.
4. Upon *g*, the centre, describe A D B, it will be the segment required.

PROBLEM

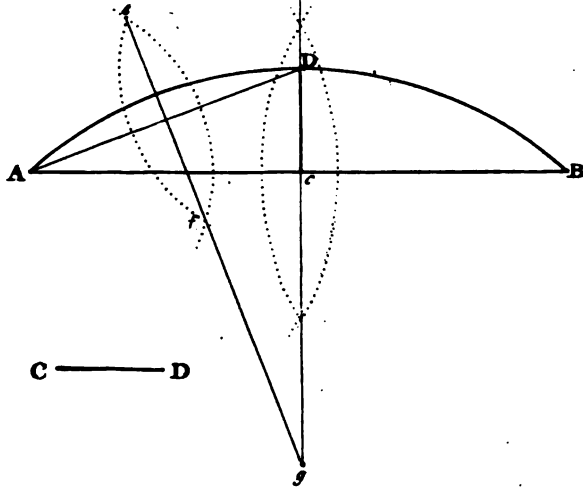
PROB. XIV.

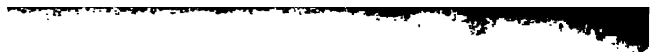


PROB. XV.



PROB. XVI.







PROB. XVII.

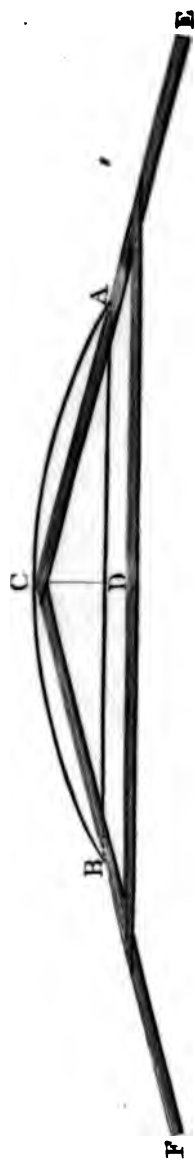


Fig. 2.



## PROBLEM XVII.

To describe the segment of a circle, by means of two rules, to any length  $AB$ , and perpendicular height  $CD$ , in the middle of  $AB$ , without making use of the centre.

*It will be most convenient for practice to make the rules  $CE$  and  $CF$  each equal to  $AB$ , as room is sometimes wanted.*

1. Place the rules to the height at  $C$ , bring the edges close to  $A$  and  $B$ , tack them together at  $C$ , and fix a rod across them to keep them tight.
2. Put in pins at  $A$  and  $B$ , then move your rules round these pins, hold a pencil to the angular point at  $C$ , it will describe the segment required.



FIG. II. *By means of a triangle, let  $AB$  be the length of the segment, and  $CD$  the perpendicular height in the middle.*

1. Through the points  $D$  and  $B$ , draw  $DB$ .
2. Draw  $DE$  parallel to  $AB$ , for conveniency, make  $DE$  equal to  $DB$ , and join  $EB$ .
3. Make a triangle  $E, D, B$ , put in pins at the points  $A, D, B$ , then move your triangle round the points  $D$  and  $B$ , and the angular point will describe half the segment; the other half will be described in the same manner, which will complete the whole segment, as was required.

FIG. III. *Another method by means of points; let A B be the length, and C D bisecting A B perpendicular, the height.*

1. Through D, draw G H parallel to A B.
2. Draw D B, the half chord.
3. From B, make B H perpendicular to D B, cutting G H in H, make D G equal to D H.
4. Draw A F and B E, each perpendicular to A B, cutting G H in F and E.
5. Divide D G, D H; C A, C B; and A F, B E; each into a like number of equal parts, as five.
6. Draw the cross lines 4 4, 3 3, 2 2, 1 1, &c.
7. From the division on A F, and B E, draw lines to D, cutting the other cross lines at *d, e, f, g*, &c.
8. Put pins in these points, bend a slip round them, and draw the curve by it, it will be the segment required.

---

FIG. IV. *Another method, by points nearly true, when the segment is very flat, let A B be the length, and C D bisecting A B, the perpendicular height.*

1. Draw A E, and B F, perpendicular to A B, each equal to C D.
2. Divide C B, and C A, each into the same number of equal parts, as 5.
3. From the points 4, 3, 2, 1, &c. on A B, draw the perpendicular 4 4, 3 3, 2 2, 1 1, &c. to A B.
4. Divide A F, and B E, into five equal parts each.
5. Draw lines from the points 1, 2, 3, 4, 5, at each end, to D, and complete the segment in the same manner as FIG. III.

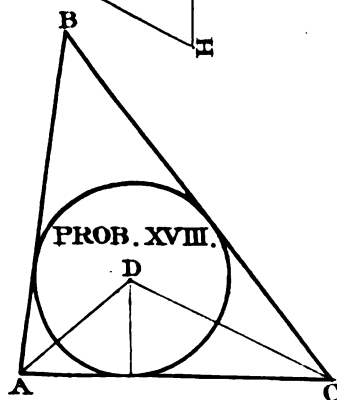
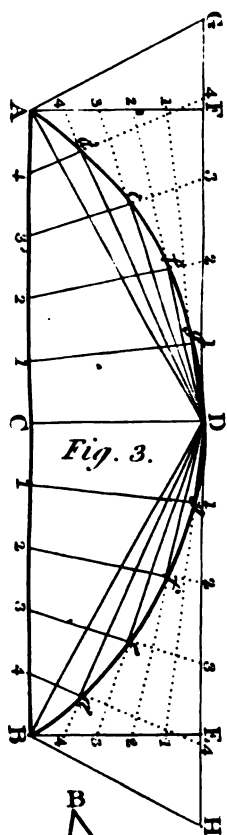
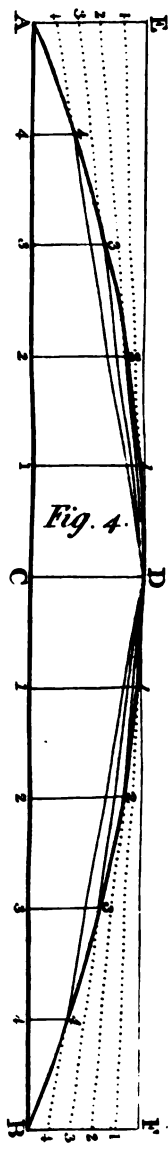
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### PROBLEM XVIII.

In a given triangle A, B, C, to inscribe a circle.

1. Bisect any two angles A and C, with the lines A D, and C D.
2. From D, the point of intersection, let fall the perpendicular D E, it will be the radius of the circle required.

PROBLEM

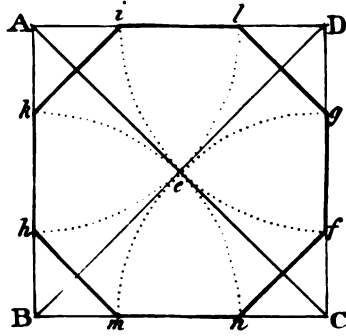




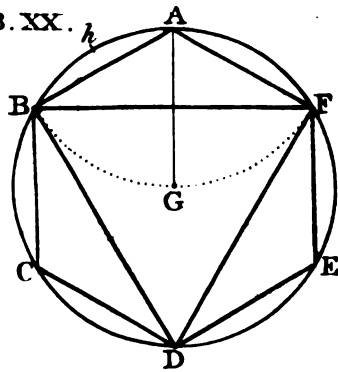


PROB. XIX.

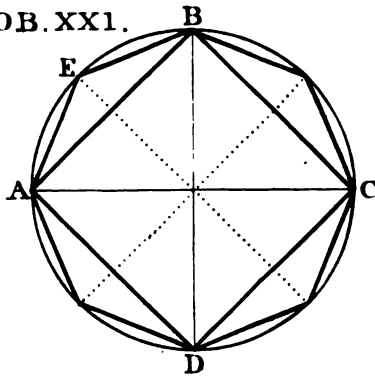
Pl. 12.



PROB. XX.



PROB. XX1.



## PROBLEM XIX.

In a given square  $A B C D$ , to inscribe a regular octagon.

1. Draw the diagonals  $A C$ , and  $B D$ , intersecting at  $e$ .
2. Upon the points  $A, B, C, D$ , as centres, with a radius  $e C$ , describe arcs  $h e l, k e n, m e g, f e i$ .
3. Join  $f n, m l, k i, h g$ , it will be the octagon required.

## PROBLEM XX.

In a given circle to inscribe an equilateral triangle, an hexagon, or a dodecagon.

*For the equilateral triangle.*

1. Upon any point  $A$ , in the circumference with the radius  $A G$ , describe the arc  $B G F$ .
2. Draw  $B F$ , make  $B D$  equal to  $B F$ .
3. Join  $D F$ , and  $B D F$  will be the equilateral triangle required.

*For the hexagon.*

Carry the radius  $A G$  six times round the circumference, the figure  $A B C D E F$  will be the hexagon.

*For the dodecagon.*

Bisect the arc  $A B$  in  $h$ , and  $A h$  being carried twelve times round the circumference, will also form the dodecagon.

## PROBLEM XXI.

In a given circle to inscribe a square or an octagon.

1. Draw the diameters  $A C$  and  $B D$ , at right angles.
2. Join  $A B, B C, C D, D A$ , and  $A B C D$  will be the square.

*For the octagon.*

Bisect the arc  $A B$  in  $E$ , and  $A E$  being carried eight times round, will also form the octagon.

PROBLEM

### PROBLEM. XXII.

In a given circle to inscribe a pentagon, or a decagon.

*For a pentagon.*

1. Draw the diameters A C and B D, at right angles.
2. Bisect E C in *f*, upon *f*, with the distance of *f* D describe the arc D *g* upon D, with the distance D *g*, describe the arc *g* H cutting the circle in H.
3. Join D H, and carry it round the circle five times, will form the pentagon.

*For the decagon.*

Bisect the arc D H in *i*, and D *i* being carried ten times round, will also form the decagon.

### PROBLEM XXIII.

In a given circle to inscribe any regular polygon.

1. Draw the diameter A B, from E the centre, erect the perpendicular E F C, cutting the circle at F.
2. Divide E F into four equal parts, and set three parts from F, to C.
3. Divide the diameter A B into as many equal parts as the polygon is required to have sides.
4. From C, through the second division in the diameter, draw C D.
5. Join A D, it will be the side of the polygon required.

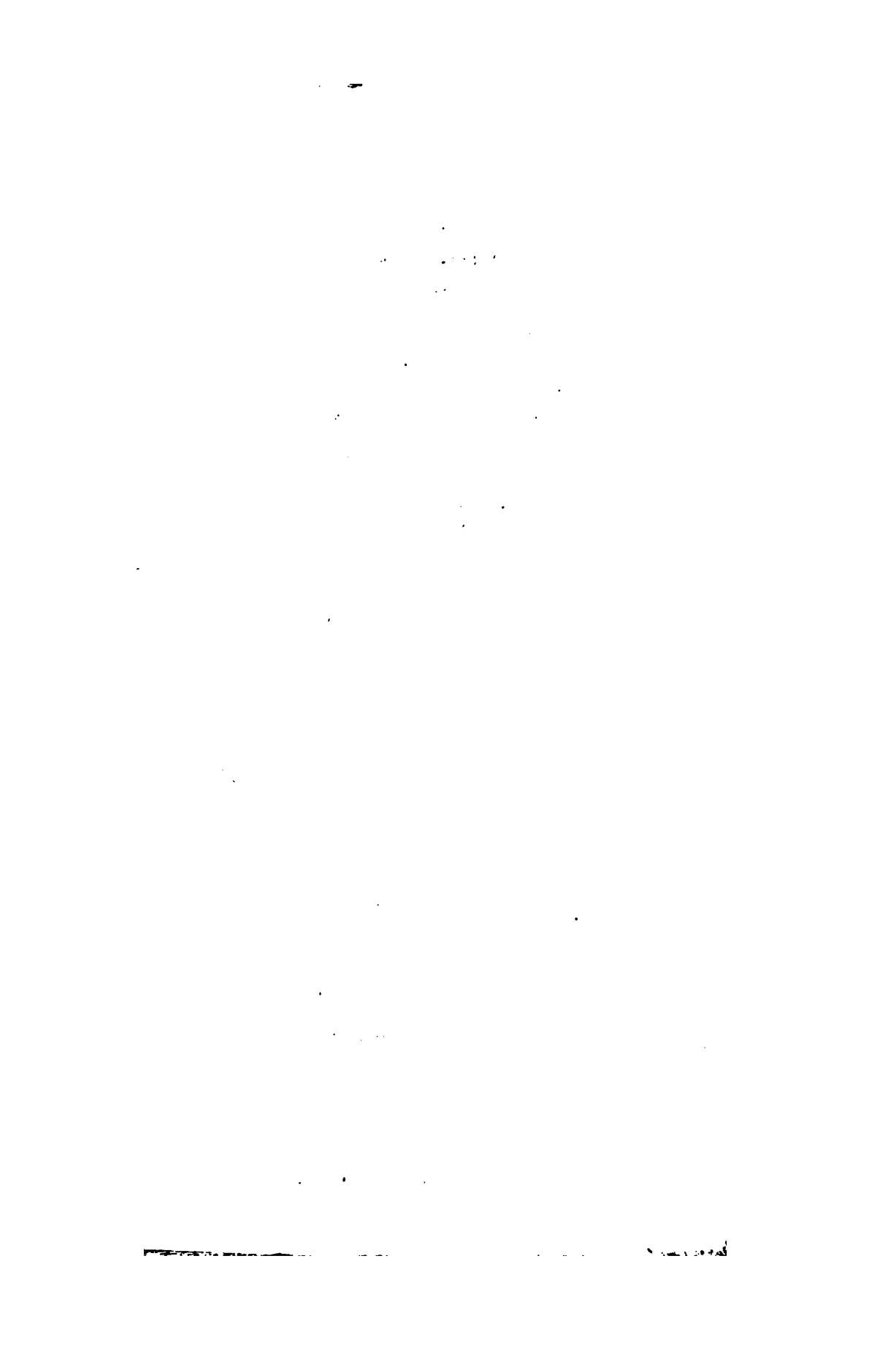
### PROBLEM XXIV.

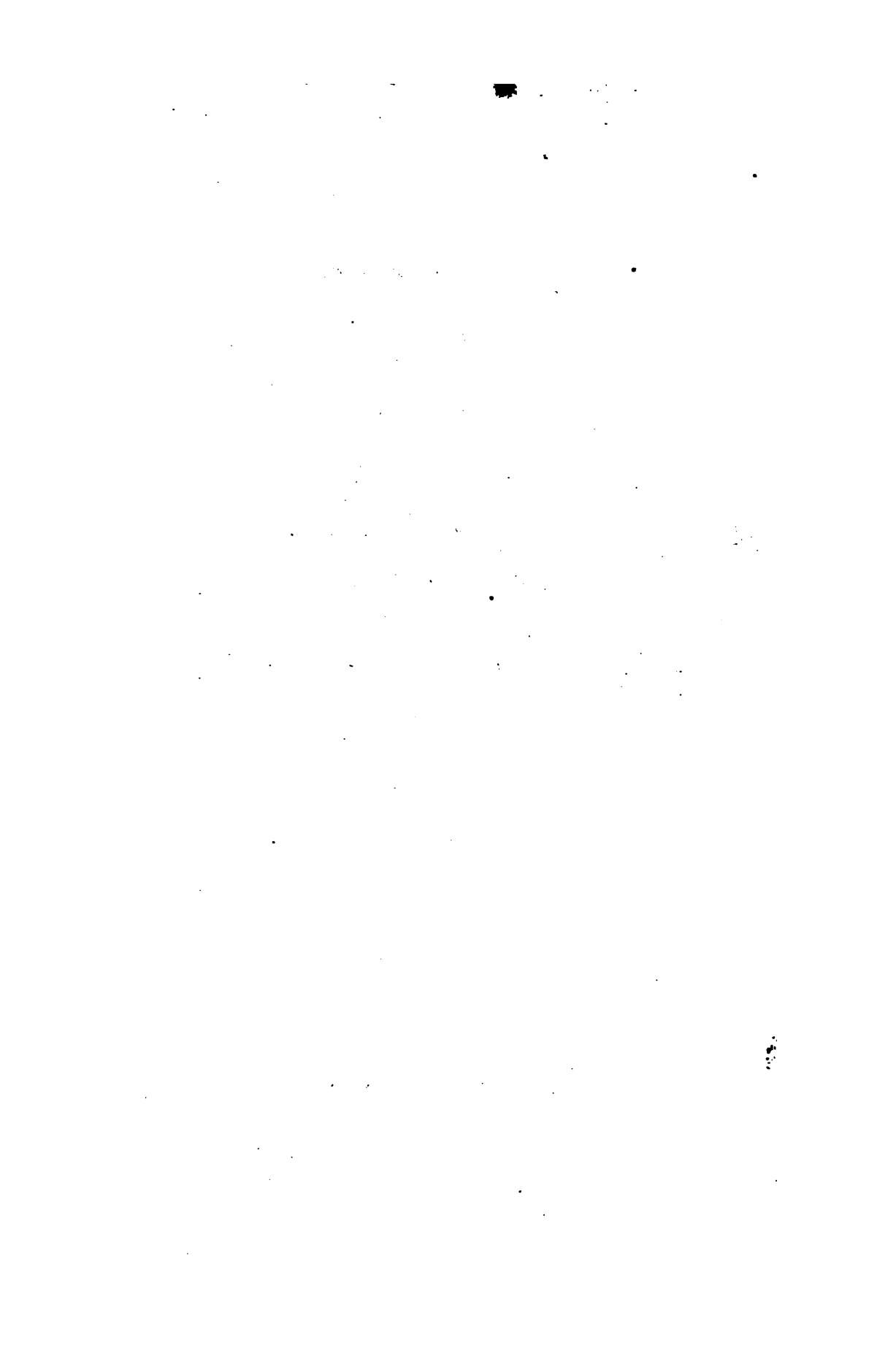
Upon a given line A B, to describe an equilateral triangle.

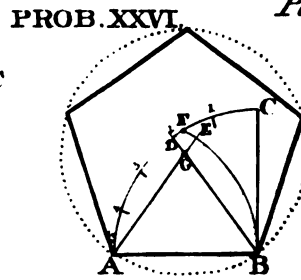
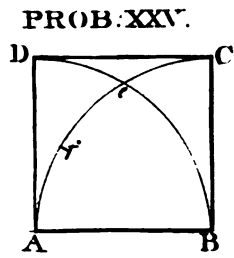
1. Upon the points A and B, with a radius equal to A B, describe arcs, cutting each other at C.
2. Draw A C and B C, it will be the triangle required.

PROBLEM

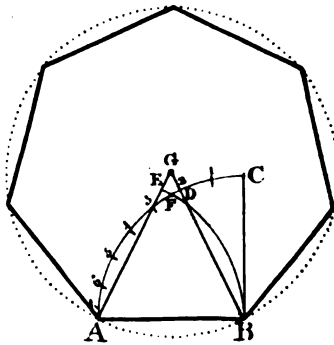




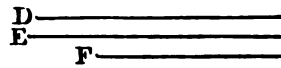
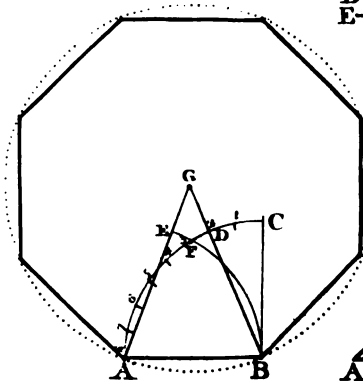




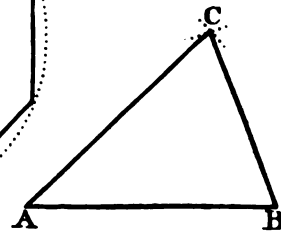
PROB. XXVI.



PROB. XXVI.



PROB. XXVII.



## PROBLEM XXV.

Upon a given line  $A B$  to describe a square.

1. Upon  $A$  and  $B$ , as centres with a radius  $A B$ , describe two arcs  $A e C$ ,  $B e D$ , cutting each other at  $e$ .
2. Bisect  $A e$  at  $f$ , from  $e$  make  $e D$  and  $e C$  equal to  $e f$ .
3. Join  $A D$ ,  $D C$ ,  $C B$ , it will be the square required.

## PROBLEM XXVI.

Upon a given line  $A B$ , to construct any regular polygon.

1. Upon  $A$  and  $B$ , as centres with a radius  $A B$ , describe two arcs intersecting each other at  $F$ .
2. From  $B$ , draw  $B C$  perpendicular, and divide the arc  $A C$  into as many equal parts as the polygon is to have sides.
3. Through the second division  $D$ , draw  $B G$ , make  $F E$  equal to  $F D$ , and through  $E$ , draw  $A G$ , meeting  $B G$  at  $G$ , then  $G$  will be the centre, and  $G A$  the radius of a circle, that will contain  $A B$  to any number of sides required.

## PROBLEM XXVII.

To make a triangle, whose three sides shall be equal to three given lines  $D$ ,  $E$ ,  $F$ , if any two are greater than the third.

1. Draw  $A B$  equal to the line  $D$ .
2. Upon  $B$ , with the length of  $E$ , describe an arc at  $C$ .
3. Upon  $A$ , with the length  $F$ , describe another arc, intersecting the former at  $C$ .
4. Draw  $A C$  and  $C B$ , and  $A B C$  will be the triangle required.

PROBLEM

### PROBLEM XXVIII.

To make a trapazium equal, and similar to a given trapazium A B C D.

1. Divide the given trapazium A B C D into two triangles, by a diagonal A C.
2. Make E F equal to A B upon E F, construct the triangle E F, whose three sides will be respectively equal to the triangle A B C.
3. Upon E G, which is equal to A C, construct the triangle E G H, whose two sides E H, and G H, are respectively equal to A D and C D, then E F G H will be the trapazium required.

In the same manner may any irregular polygon be made equal and similar to a given irregular polygon, by dividing the given polygon into triangles, and constructing the triangles in the same manner in the required polygon, as is shown by figures.

### PROBLEM XXIX.

To make a triangle equal to a given trapazium A B C D.

1. Draw the diagonal B D, make C E parallel to it, meeting the side A B, produced in E.
2. Join D E, and A D E will be the triangle.

### PROBLEM XXX.

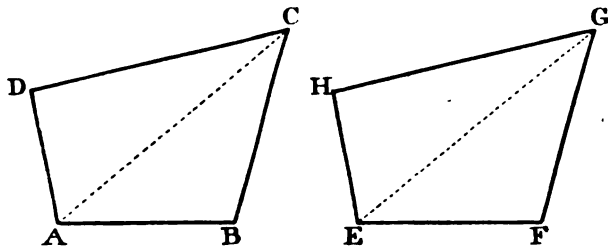
To make a triangle equal to any given right-lined figure A B C D E.

1. Produce the side A B both ways at pleasure.
2. Draw the diagonals A D and B D, and make E F and G H parallel to them.
3. Join D F, D G, then D F G will be the triangle required.

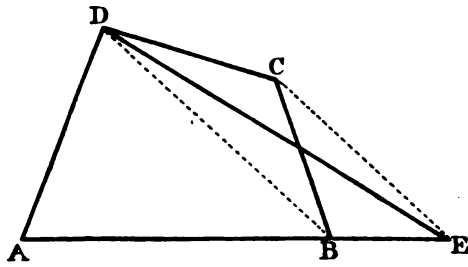
Much after the same manner may any other right-line figure be reduced to a triangle.

PROBLEM

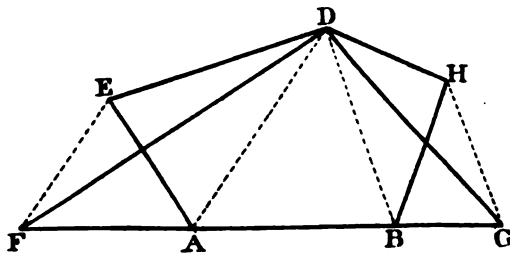
**PROB. XXVIII.**



**PROB. XXIX.**



**PROB. XXX.**

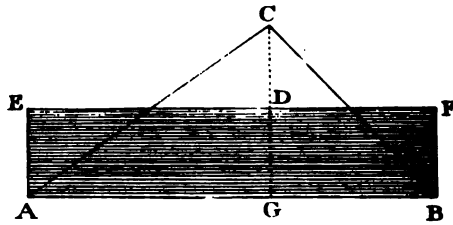




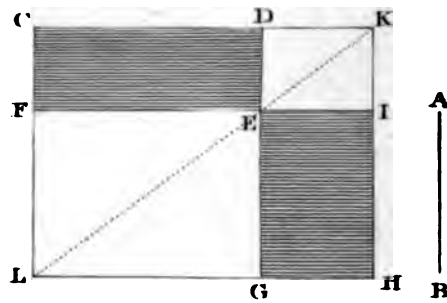


PROB. XXXI.

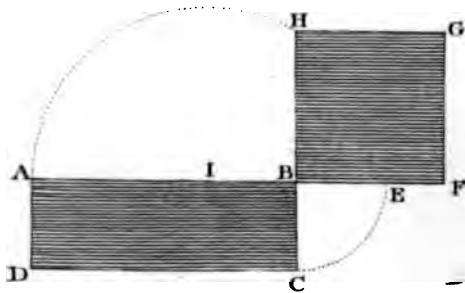
Pl. 16.



PROB. XXXII.



PROB. XXXIII.



## PROBLEM XXXI.

To reduce a triangle  $ABC$  to a rectangle.

1. Bisect the altitude  $CG$  in  $D$ , through  $D$  draw  $EF$  parallel to  $AB$ .
2. From  $B$  draw  $BF$  perpendicular to  $AB$ , through  $A$  draw  $AF$  parallel to  $BF$ , then  $ABFE$  will be the rectangle required.

## PROBLEM XXXII.

To make a rectangle, having a side equal to a given line  $AB$ , and equal to a given rectangle  $CDEF$ .

1. Produce the sides of the rectangle  $CF$ ,  $DE$ ,  $FE$ , and  $CD$ .
2. Make  $EG$  equal to  $AB$ , through  $G$  draw  $LH$  parallel to  $FE$ , cutting  $CF$  produced at  $L$ .
3. Draw the diagonal  $LE$ , and produce it till it cut  $CD$  at  $K$ .
4. Draw  $KH$  parallel to  $EG$ , then will  $EIHG$  be the rectangle required.

## PROBLEM XXXIII.

To make a square equal to a given rectangle  $ABCD$ .

1. Produce the side  $AB$ , make  $BE$  equal to  $BC$ .
2. Bisect  $AE$  in  $I$ , on  $I$ , as a centre with the radius  $IE$  or  $IA$ , describe the semicircle  $AHE$ .
3. Produce the side of  $CB$  to cut the circle in  $H$ , on  $BH$  describe the square  $BHGF$ , it will be the square required.

### PROBLEM XXXIV.

To make a square equal to two given squares  
A and B.

1. Make D E equal to the side of the square A, and D F perpendicular to D E, equal to the side of the square B.
2. Draw the hypotenuse F E; on it describe the square E F G H, it will be the square required.

### PROBLEM XXXV.

To make a square equal to three given squares  
A, B, C.

1. Make D E equal to the side of the square A, and D F perpendicular to D E, equal to the side of the square B.
2. Join F E, draw F G perpendicular to it.
3. Make F G equal to the side of the square C; join G E, then G E will be the side of the square required.

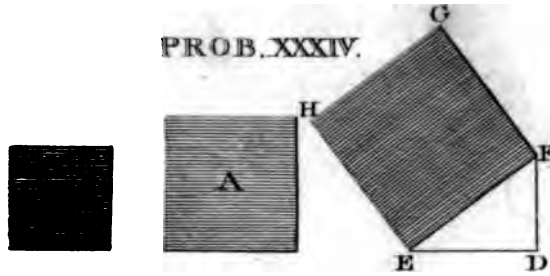
### PROBLEM XXXVI.

Two right lines A B, and C D. being given, to  
find a third proportional.

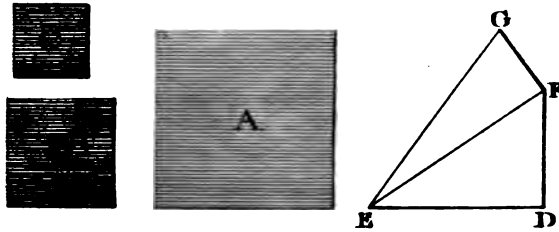
1. Make an angle H E I at pleasure, from E, make E F equal to A B, and E G equal to C D, join F G.
2. Make E H equal to E G, and draw H I parallel to F G, then E I will be the third proportional required, that is,  $E F : E G :: E H : E I$ , or  $A B : C D :: C D : E I$ .

PROBLEM

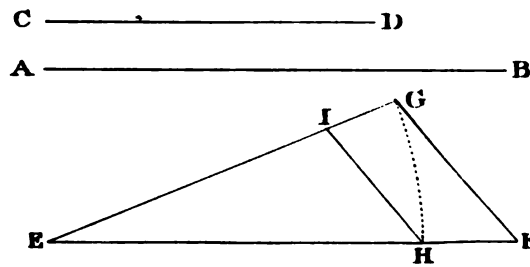
PROB. XXXIV.



PROB. XXXV.



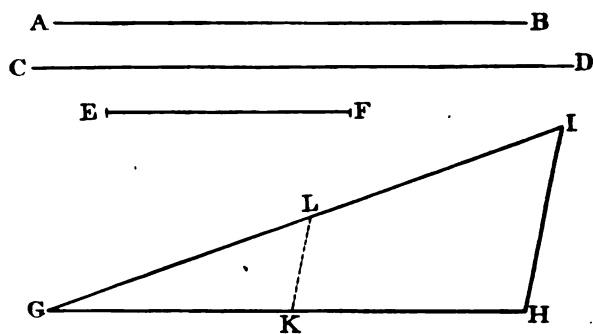
PROB. XXXVI.



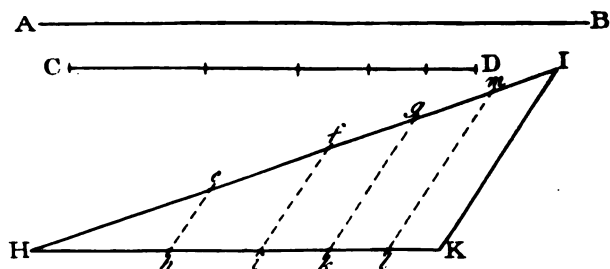




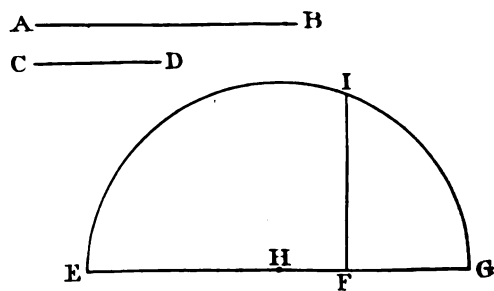
PROB. XXXVII. *Pl. 18.*



PROB. XXXVIII.



PROB. XXXIX.



## PROBLEM XXXVII.

Three right lines  $AB$ ,  $CD$ ,  $EF$ , being given,  
to find a fourth proportional.

1. Make the angle  $HGI$  at pleasure; from  $G$  make  $GH$  equal to  $AB$ ;  $GI$  equal to  $CD$ ; and join  $HI$ .

2. Make  $GK$  equal to  $EF$ , draw  $KL$  through  $K$  parallel to  $HI$ , then  $GL$  will be the fourth proportional required; that is  $GH : GI :: GK : GL$ , or  $AB : CD :: EF : GL$ .

## PROBLEM XXXVIII.

To divide a given line  $AB$ , in the same proportion as another,  $CD$ , is divided.

1. Make any angle  $KHI$ ; and make  $HI$  equal to  $AB$ ; then apply the several divisions of  $CD$  from  $H$  to  $K$ , and join  $KI$ .

2. Draw the lines  $he, if, kg$ , parallel to  $IK$ , and the line  $HI$ , will be divided in  $h, i, k$ , as was required.

## PROBLEM XXXIX.

Between two given right lines,  $AB$  and  $CD$ ,  
to find a mean proportional.

1. Draw the right line  $EG$ , in which make  $EF$  equal to  $AB$ , and  $FG$  equal to  $CD$ .

2. Bisect  $EG$  in  $H$ , and with  $HE$  or  $HG$ , describe the semicircle  $EIG$ .

3. From  $F$  draw  $FI$  perpendicular to  $EG$ , cutting the circle in  $I$ , and  $IF$  will be the mean proportional required.

## PROBLEM XL.

To find a line nearly equal to the circumference of its circle  $ACBD$ .

1. Draw the diameters  $AB$  and  $CD$  at right angles.
2. Produce  $AB$ , till the part  $AG$  without, be three quarters of the radius.
3. Draw  $EF$  through  $B$ , parallel to  $CD$ ; through  $G$ , and the points  $C$  and  $D$ ; draw  $GE$  and  $GF$ , cutting the tangent in  $E$  and  $F$ ; then  $E$  and  $F$  will be equal to half the circumference.

Much after the same manner may a straight line be found equal to any part of a circle, as is shown at Fig. 2, but the following method is much better for small arcs; as it requires less room.

## REMARK.

*If any number of divisions  $E I, I K, K L, L B$ , are taken on  $EF$ , and from the points  $I, K, L$ , lines are drawn to  $G$ , to cut the circumference  $i, k, l$ , the divisions on the circle, viz.  $C i, i k, k l, l B$ , will be respectively equal to their corresponding divisions  $E I, I K, K L, L B$ , on the tangent line  $EF$ ; that is,  $BL$  will be equal to  $B l$ , and  $L K$  equal to  $l k$ ,  $K I$  equal to  $k i$ , and  $I E$  equal to  $i C$ .*

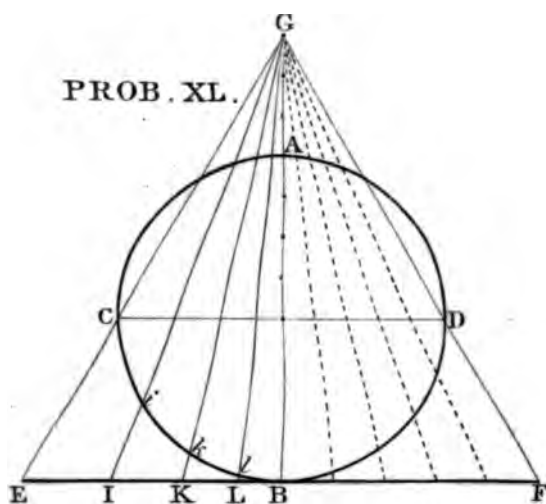
## PROBLEM XLI.

To find the length of any arc  $ACB$  of a circle.

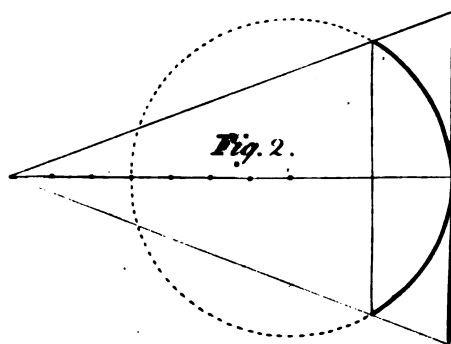
1. Draw the chord  $AB$  indefinitely towards  $E$ , and bisect the arc  $ACB$  at  $C$ .
2. Make  $AD$  equal to twice the half chord  $AC$ ; divide  $BD$  into three equal parts, and set one towards  $E$ ; then will  $AE$  be the length of the arc line  $ACB$ .

CONIC

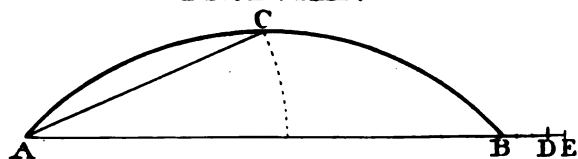
PROB. XL.



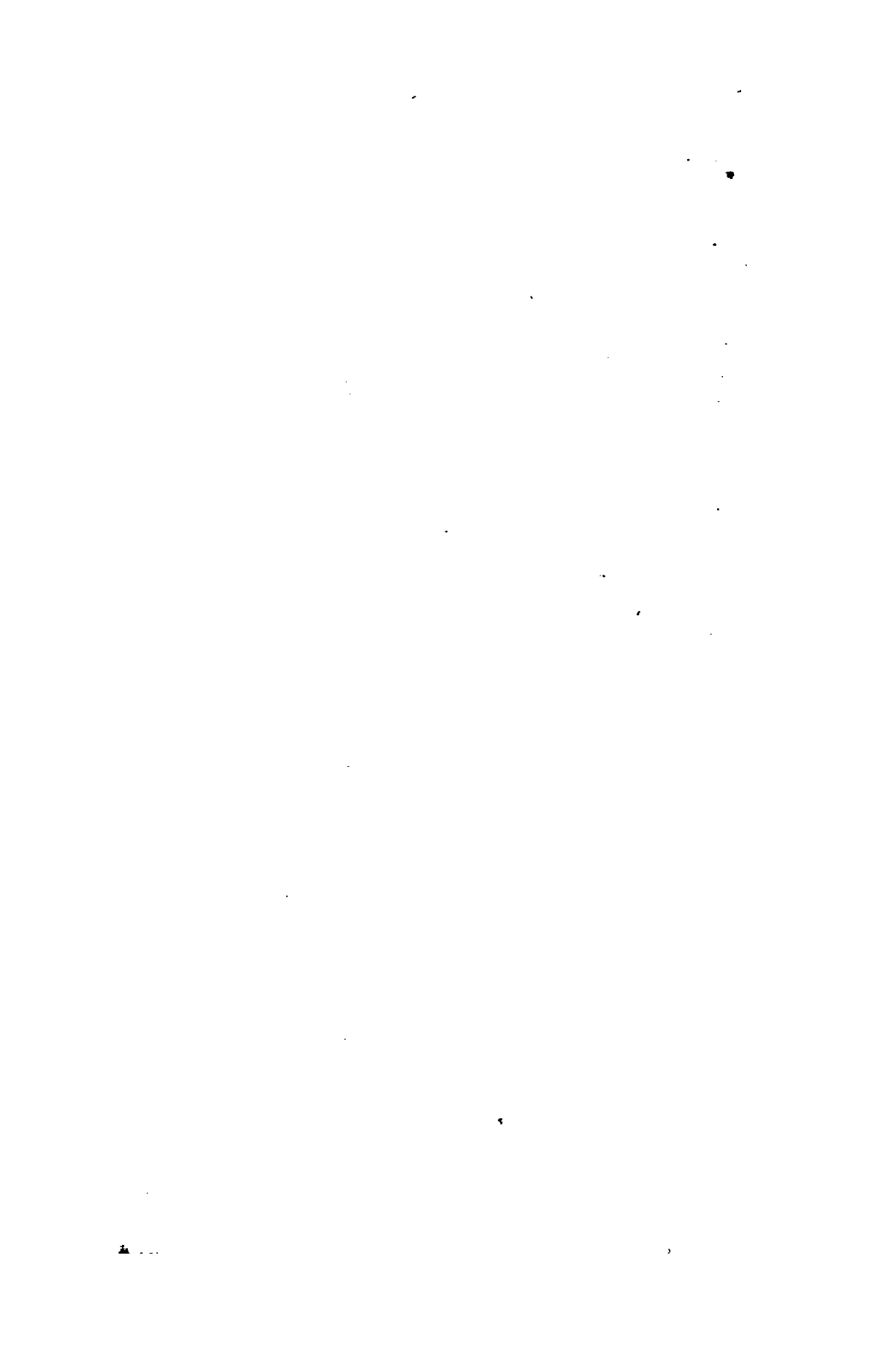
*Fig. 2.*



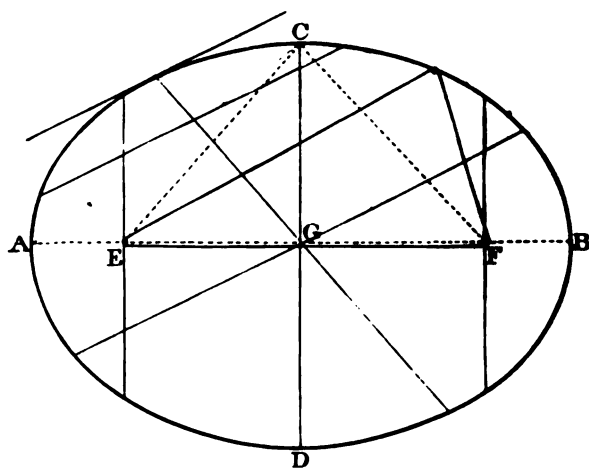
PROB. XLI.







*Definitions*



## CONIC SECTIONS.

### OF THE ELLIPSIS.

#### DEFINITIONS.

1. If two pins are fixed at the points E and F, a string being put about them, and the ends tied together at C; the point C being moved round, keeping the string stretched, it will describe a curve called an *Ellipsis*.
2. *Foci*, are the two points E and F, about which, the string is made to revolve.
3. *Transverse axis*, is the line A B, passing through the foci, and terminated by the curve at A and B.
4. *Centre*, is the point G, bisecting the transverse axis A B.
5. *Conjugate axis*, is the line C D, bisecting the transverse axis at right angles, and terminated by the curve.
6. *Latus rectum*, is a right line passing through the focus F, at right angles to the transverse axis terminated by the curve; this is also called the *Parameter*.
7. *Diameter* is any line passing through the centre G, terminated by the curve.
8. *Conjugate diameter*, is a right line drawn through the centre, parallel to a tangent at the extreme of the other diameter, and terminated by the curve.
9. *Double ordinate*, is a line drawn through any diameter, parallel to a tangent at the extreme of that diameter, terminated by the curve.

#### PROBLEM

### PROBLEM I.

The transverse and conjugate axes  $AB$ , and  $CD$ , of an ellipsis being given, to find the two foci, from thence to describe an ellipsis.

1. Take the semi transverse  $AE$ , or  $EB$ , and from  $C$ , as a centre, describe an arc, cutting  $AB$ , at  $F$  and  $G$ , which are the foci.

2. Fix pins in these points, a string being stretched about the points  $FCG$ , then move the point  $C$  round the fixed points  $F$  and  $G$ , keeping the string tight it will describe the ellipsis, as in the first definition.

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### PROBLEM II.

The same being given, as in the last problem, to describe an ellipsis, by an instrument called a trammel.

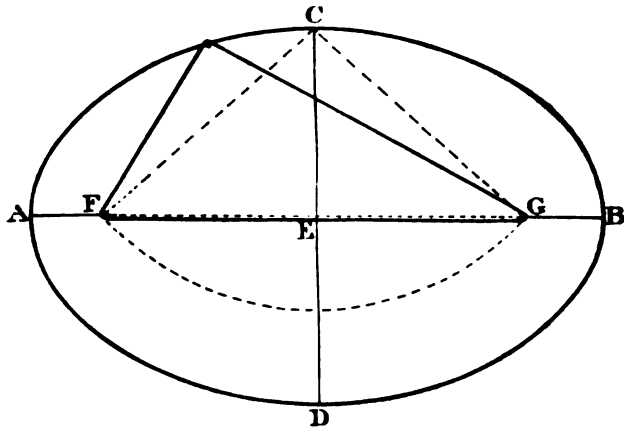
*The trammel, as is used by artificers, is two rules, with a groove in each, fixed together, so that the grooves will be at right angles to each other; to this, there is a rod, with two moveable nuts, and another fixed at the end, with a hole through it to hold a pencil; on the under side of the sliding nuts are two round pins, made to fill the groove of the trammel, and is used as follows.*

#### OPERATION.

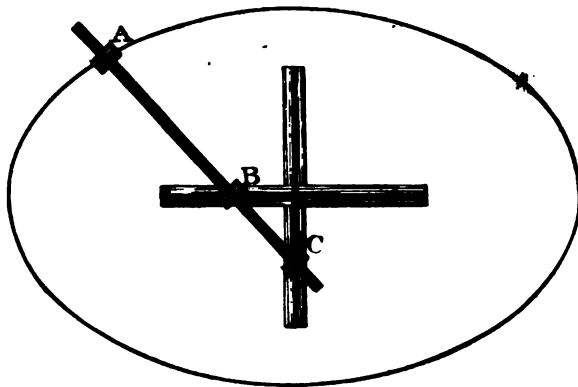
Set the distance of the first pin at  $B$ , from the pencil at  $A$ , to half the shortest axis, and the distance of the second pin at  $C$ , from  $A$ , to half the longest axis, the pins being put in the grooves, as is shown by the figure, then move the pencil at  $A$ , it will describe the ellipsis required.

### PROBLEM

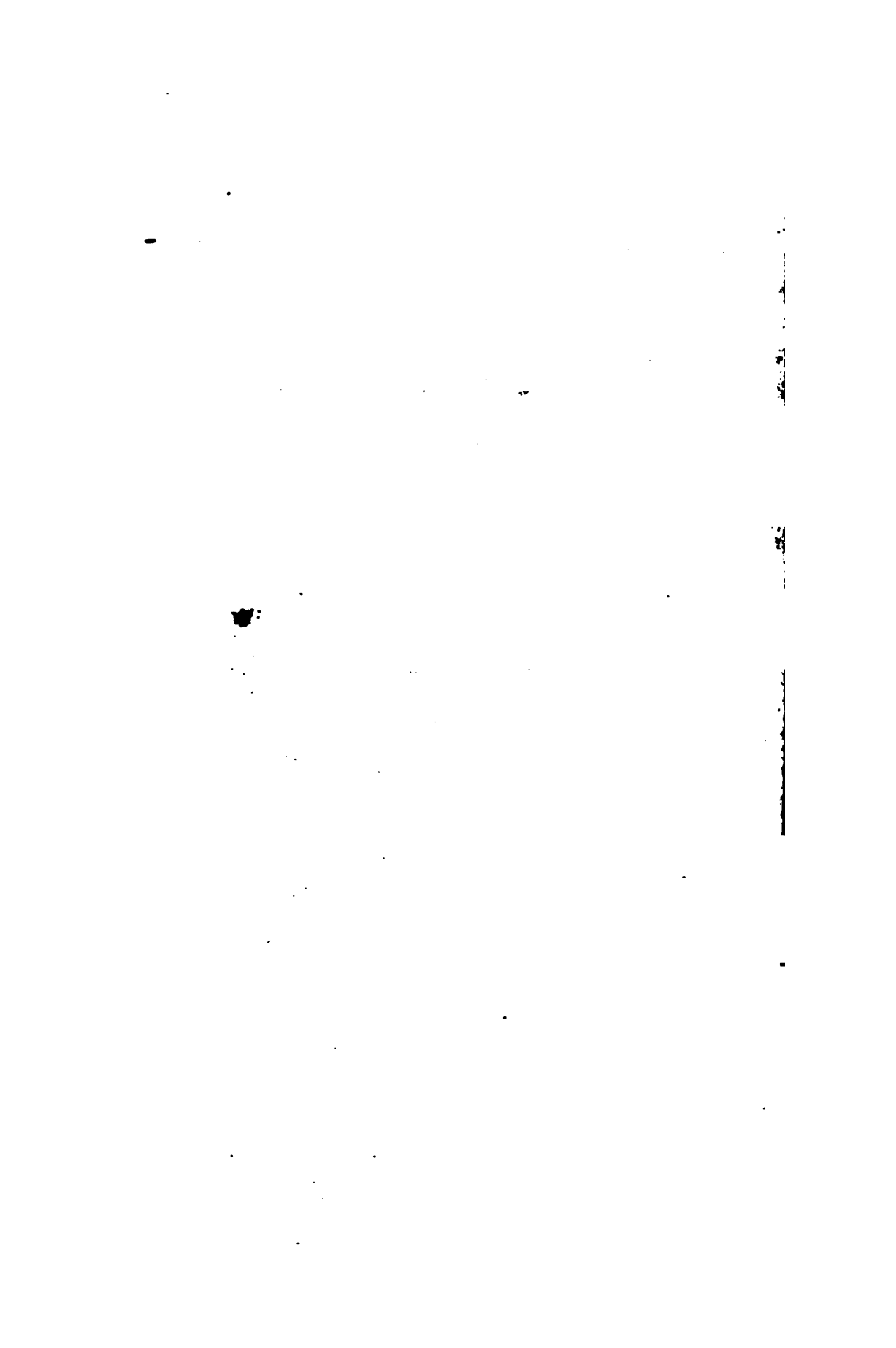
PROB. I.

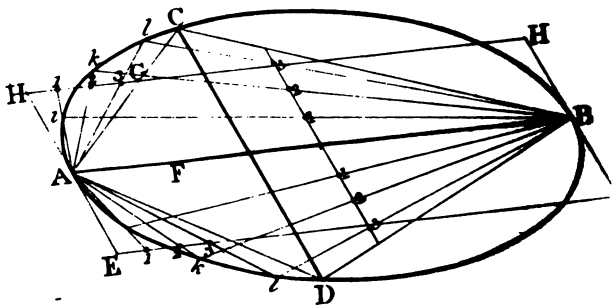
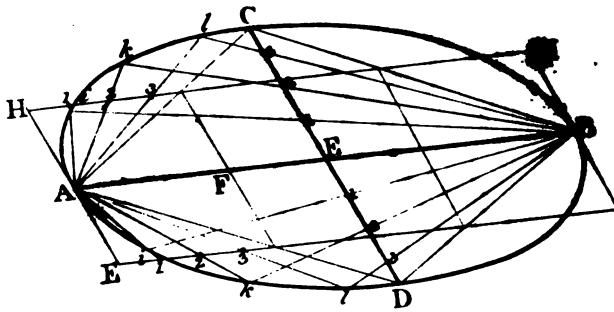
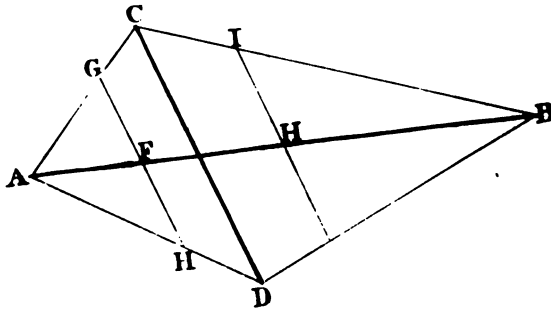


PROB. II.









## PROBLEM III.

A diameter  $AB$ , and a double ordinate  $CD$  to that diameter, being given, to find the parameter.

1. Join  $AC$ ,  $AD$ , and  $BC$ ,  $BD$ ; bisect  $AB$  in  $H$ , through  $H$  draw  $HI$  parallel to  $DC$ , cutting  $BC$  in  $I$ .
2. From  $A$ , make  $AF$  equal to  $HI$ , through  $F$  draw  $GH$  parallel to  $CD$ , cutting  $AC$  in  $G$ , and  $AD$  in  $H$ , then  $GH$  is the parameter sought.

## PROBLEM IV.

To describe an ellipsis by finding points in the curve, having the two conjugate diameters  $AB$ , and  $CD$  given.

1. Find  $FG$  half the parameter; through  $G$ , draw  $HH$  parallel to  $AB$ .
2. Draw  $EH$  parallel to  $CD$ , cutting  $HH$ , at  $H$ .
3. Set off any number of equal divisions, from  $H$ , towards  $G$ , set the same parts from  $E$ , towards  $C$ .
4. From the point  $B$ , through the points 1, 2, 3, in  $EC$ , draw the lines  $Bi$ ,  $Bk$ ,  $Bl$ .
5. From  $A$ , through the points in  $HG$ , draw the lines  $Ai$ ,  $Ak$ ,  $Al$ , intersecting the former lines in  $i$ ,  $k$ ,  $l$ , they will be in the periphery of the ellipsis.

## PROBLEM V.

Having a diameter, and a double ordinate to that diameter, to describe the ellipsis, by finding points in the curve.

This problem will be completed in the same manner as in the last problem, as is plainly shewn by the figure.

PROBLEM

## PROBLEM VI.

To describe an ellipsis, or any segment of an ellipsis, having a diameter and a double ordinate, by means of points being found in the curve, without finding the parameter.

*Let A B be a diameter or double ordinate, let C D be its conjugate, and let E D be the height of the segment.*

1. Through D, draw F G parallel to A B; also through the points A and B, draw A F, and B G, parallel to D E, cutting F G, in F and G.

2. Divide A E and E B into a like number of equal parts, as four; likewise B G, and A F, into the same number of equal parts.

3. From the point D, through the points 1, 2, 3, in A F, and B G, draw 1 D, 2 D, 3 D.

4. From the point C, through the points 1, 2, 3, in A B, draw C a, C b, C c, cutting the lines 1 D, 2 D, 3 D, in a, b, c, they will be in the periphery of the ellipsis; a curve being traced through these points, will form the ellipsis required.

But if the curve is very large, as in practical works; the best way is to put in nails or pins at the points a, b, c, &c, bend a slip round them, and draw a curve by it, it will appear quite regular.

## PROBLEM



1. The first part of the document is a list of names and addresses of the members of the committee.



PROB. VII.

Pl. 24.

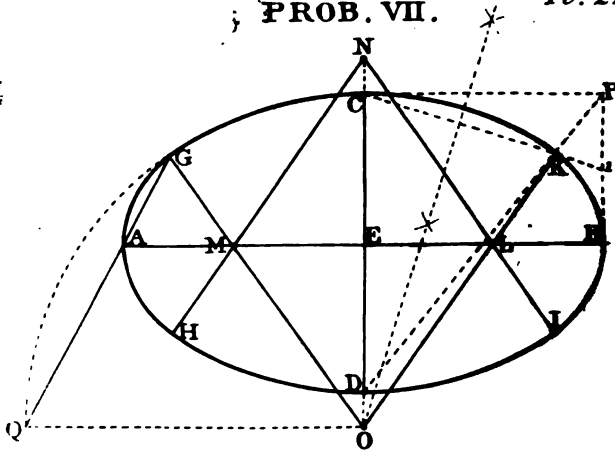
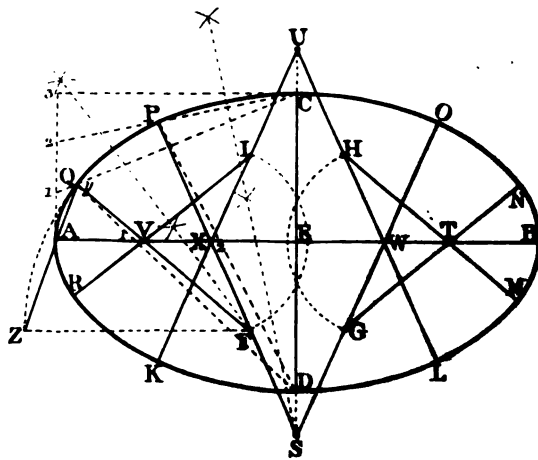


Fig. 2 .



## PROBLEM VII.

To draw the representation of an ellipses, with a compass to any length  $AB$ , and width  $CD$ .

1. Draw  $BP$  parallel and equal to  $EC$ , and bisect it at  $I$ , then draw  $IC$  and  $PD$ , cutting each other at  $K$ ; bisect  $KC$  by a perpendicular, meeting  $CD$  at  $O$ ; and on  $O$ , with the radius  $OC$ , describe the quadrant  $CGQ$ .

4. Through  $Q$  and  $A$ , draw  $QG$ , cutting the quadrant at  $G$ ; then draw  $GO$ , cutting  $AB$  at  $M$ : make  $EL$  equal to  $EM$ ; also  $EN$  equal to  $EO$ . From  $O$ , through  $M$  and  $L$ , draw  $OG$ , and  $OK$ ; likewise from  $N$ , through  $M$  and  $L$ , draw  $NH$  and  $NI$ ; then  $M, L, N, O$ , are the four centres: by help of these the four opposite sectors will be described.

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FIG. II. To describe an ellipsis more accurately with a compass than the foregoing, having the two axes  $AB$  and  $CD$  given.

1. Draw  $AS$  parallel and equal to  $EC$ , divide it into three equal parts and draw  $2C$  and  $1C$ ; then divide  $AE$  also into three equal parts, and from  $D$ , through the points  $1, 2$ , in  $AE$ , draw  $DQ$ , and  $DP$ , cutting the lines  $1C$  and  $2C$ , in  $Q$  and  $P$ .

2. Bisect  $CP$  by a perpendicular, meeting  $CD$  produced at  $S$ , join  $PS$ , cutting  $AE$  at  $X$ ; then make  $EW$  equal to  $EX$ , and  $EU$  equal to  $ES$ ; and through  $X$  and  $W$ , draw  $PS$  and  $OS$ ; also through the same points  $X$  and  $W$ , draw  $UK$  and  $UL$ .

6. Bisect  $PQ$  by a perpendicular, meeting  $PS$  at  $F$ ; draw  $ZF$  parallel to  $AB$ ; then with the radius  $FQ$  describe the arc  $QZ$ , cutting  $FZ$  at  $Z$ ; through  $Z$  and  $A$  draw  $Zy$ , cutting the arc  $ZQ$  at  $y$ ; and join  $yF$ , cutting  $AB$  at  $V$ . On  $X$ , make  $XI$  equal to  $XF$ ; with the same radius on  $W$ , make  $WH$ , and  $WG$ ; through  $V$ , draw  $IR$ , make  $ET$  equal  $EV$ , through  $T$ , draw  $HM$  and  $GN$ ; then  $U, S, G, H, I, F, T, V$ , are the centres.

## P R O B L E M . VIII.

Having the two axes, or any other conjugate diameters  $A B$  and  $C D$  given, to describe an ellipsis through points, at the extremes of any diameters taken at pleasure.

1. Through  $D$ , draw  $P Q$  parallel to  $A B$  from  $D$ ; draw  $D F$  perpendicular to  $P Q$ , and make it equal to  $E E$ , or  $E A$ ; upon  $F$ , with the distance  $F D$ , describe the circle  $\propto D k$ .

2. Through the centre  $E$ , draw the line  $P E N$ ,  $t E M$ ,  $s E L$ , &c. at pleasure, cutting the tangent  $P Q$ , at  $P$ ,  $t$ ,  $s$ , &c. Join  $P F$ ,  $t F$ ,  $s F$ , &c. cutting the circle  $\propto D k$ , at the points  $m \propto l$ , &c.; likewise join  $E F$ , if necessary, and draw  $n N$ ,  $m M$ ,  $l L$ , &c. parallel to it, cutting the diameters  $N N$ ,  $M M$ ,  $L L$ , &c. at  $N$ ,  $M$ ,  $L$ , &c. then these points will be in the periphery of the ellipsis. If the diameters are produced to the opposite sides, at  $N$ ,  $M$ ,  $L$ , and the distances  $E N$ ,  $E M$ ,  $E L$ , &c. are made respectively to their corresponding opposite distances  $E N$ ,  $E M$ , and  $E L$ , &c. then the points  $N M L$ , on the under side of the diameter  $A B$ , will also be in the curve.

## P R O B L E M IX.

To draw an ellipsis by ordinates, having the two axes, or any other conjugate diameters,  $A B$ , and  $C D$ , given.

1. From  $E$ , the centre, draw  $E F$  perpendicular to  $C D$ . Upon  $E$ , with the radius  $E C$ , describe the quadrant  $C F$ ; divide  $E F$  into any number of equal parts, as four; from these points draw  $1 a$ ,  $2 b$ ,  $3 c$ , parallel to  $E C$ , cutting the quadrant at  $a$ ,  $b$ , and  $c$ .

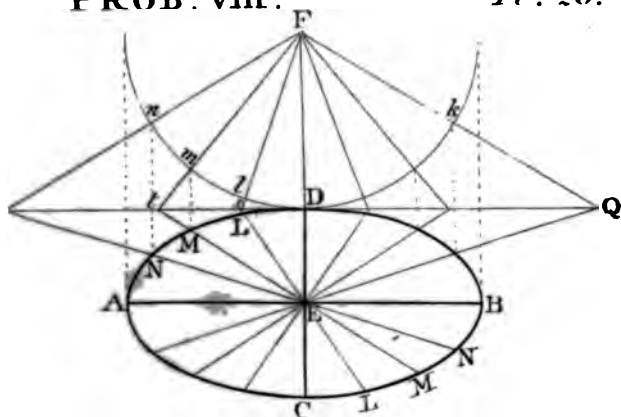
2. Divide  $E A$ , and  $E B$ , each in the same number of equal parts; through the points 1, 2, 3, &c. draw  $a a$ ,  $b b$ ,  $c c$ , &c. parallel to  $C D$ .

5. Make the distance  $1 a$ ,  $2 b$ ,  $3 c$ , &c. equal to their corresponding distances,  $1 a$ ,  $2 b$ , on the quadrant; then the points  $a$ ,  $b$ ,  $c$ , &c. will be all in the periphery of the ellipsis.

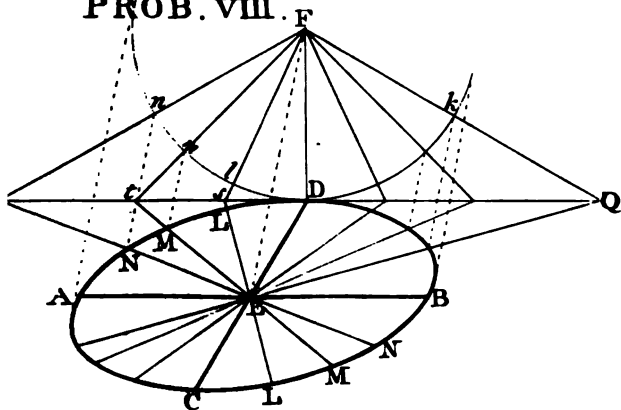
PROBLEM

PROB. VIII.

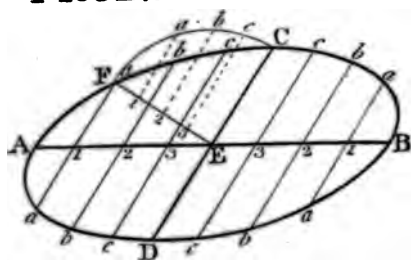
*Pl. 25.*

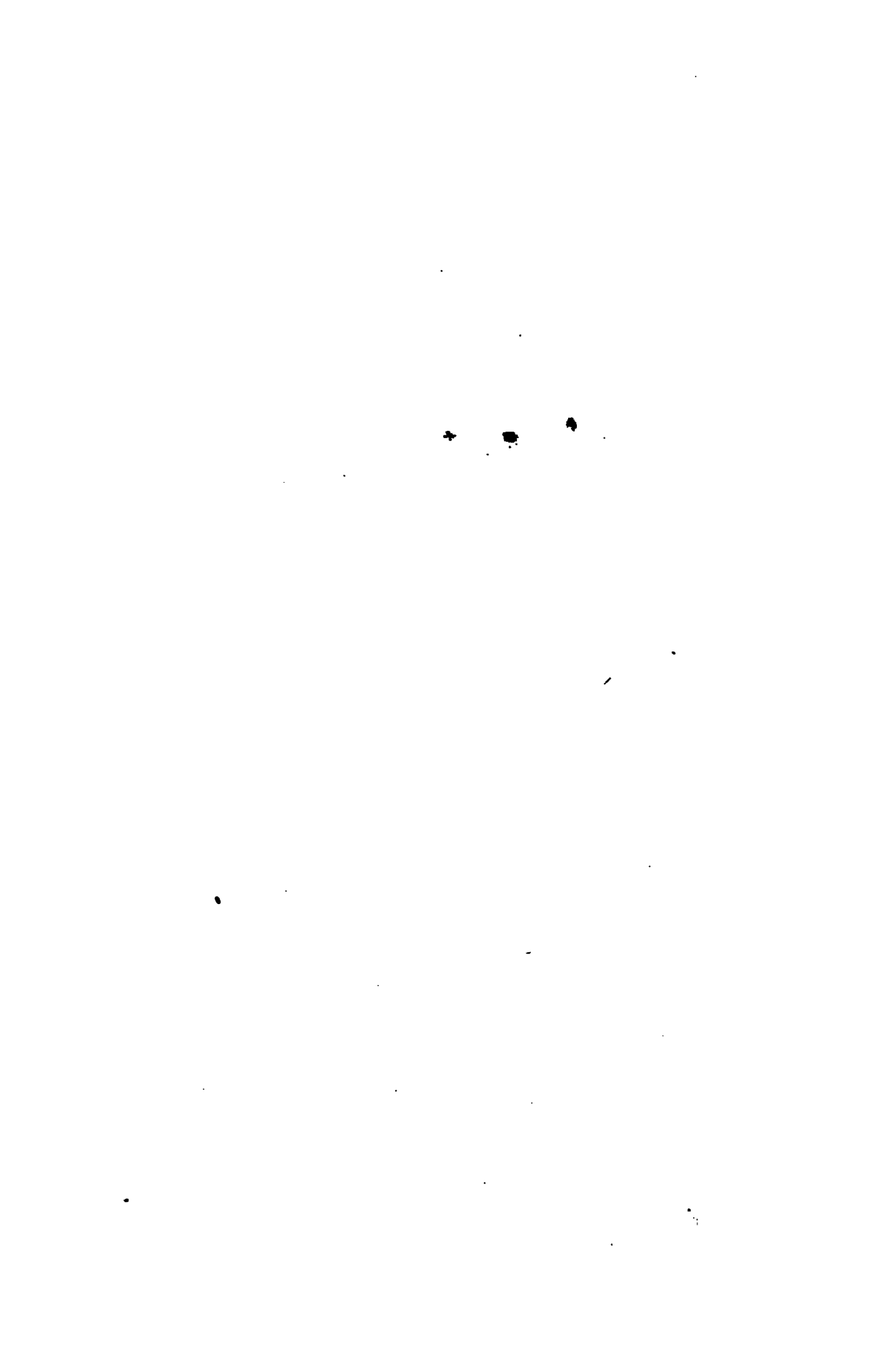


PROB. VIII.



PROB. IX.





1000

1000

1000

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1000

1000



## PROBLEM X.

An ellipsis  $A D B C$  being given, to find the transverse and conjugate axes.

1. Draw any two parallel lines  $A B$ , and  $C D$ , cutting the ellipsis at the points  $A, B, C, D$ ; bisect them in  $e$  and  $f$ .
2. Through  $e$  and  $f$  draw  $G H$ , cutting the ellipsis at  $G$  and  $H$ ; bisect  $G H$ , at  $I$ , will give the centre.
3. Upon  $I$ , with any radius, describe a circle, cutting the ellipsis in the four points  $k, l, m, n$ .
4. Join  $k l$ , and  $m n$ ; bisect  $k l$ , or  $m n$ , at  $o$  or  $p$ .
5. Through the points  $o$  or  $p$ , draw  $Q R$ , cutting the ellipsis at  $Q$  and  $R$ ; then  $Q R$  will be the transverse axis.
6. Through  $I$ , draw  $T S$  parallel to  $k l$ , cutting the ellipsis at  $T$  and  $S$ , and  $T S$  will be the conjugate axis.

## PROBLEM XI.

Any diameter  $A B$  being given, and an ordinate  $C D$ , to find its conjugate, without drawing any part of the ellipsis.

1. Draw  $C I$  perpendicular to  $A B$ ; bisect  $A B$  in  $F$ , and draw  $F H$  parallel to  $C D$ .
2. On  $F$ , with the distance  $F A$ , or  $F B$ , describe the semicircle  $A I B$ , cutting  $C I$ , at  $I$ .
3. Make  $A E$  equal to  $C I$ ; draw  $E G$  parallel and equal to  $C D$ ; through  $G$  and  $A$ , draw  $A H$ , cutting  $F H$ , at  $H$ ; then  $F H$  is the semiconjugate.

Much after the same manner; if two conjugate diameters are given, an ordinate may be found without drawing any part of the ellipsis.

### PROBLEM XII.

Any two conjugate diameters  $A B$  and  $C D$ , being given, and a right line  $G H$  passing through the centre  $F$ , to find a diameter which will be conjugate to  $G H$ , without drawing any part of the ellipsis.

1. Through  $D$ , draw  $E K$  parallel to  $A B$ , and produce the given line  $H G$ , to cut the tangent in  $E$ .
2. From  $D$ , make  $D I$  perpendicular to  $E F$ , and equal to  $F A$ , or  $F B$ .
3. Join  $E I$ ; from  $I$ , draw  $I K$  perpendicular to  $I E$ , cutting the tangent  $E K$ , at  $K$ ; through the centre  $F$ , draw  $F K$ .
4. Through the points  $g$ , and  $m$ , where the lines  $E I$ , and  $I K$ , cut the circle; draw  $g G$ , and  $m M$ , parallel to  $I F$ , cutting  $E F$ , and  $K F$ , at the points  $G$  and  $M$ ; make  $F H$ , equal to  $F G$ , and  $F L$ , equal to  $F M$ ; then  $M L$ , and  $G H$ , will be the two other conjugate diameters.

---

### PROBLEM XIII.

Any two conjugate diameters  $A B$  and  $C D$ , being given, to find the two axes, from thence, to describe the ellipsis.

1. Through  $D$ , draw  $E F$ , parallel to  $A B$ ; draw  $D I$  perpendicular to  $E F$  and equal to  $M A$ , or  $M B$ .
2. Upon  $I$ , with the radius  $I D$ , describe the arc  $g D l$ .
3. Join  $I M$ , and bisect it by a perpendicular, meeting the tangent  $E F$  at  $N$ .
4. On  $N$ , as a centre, with the distance  $N I$ , describe a semicircle  $E I F$ , cutting  $E F$ , at the points  $E$  and  $F$ .
5. Through the centre  $M$ , draw  $F K$ , and  $E H$ .
6. Join  $I F$ , and  $I E$ , cutting the arc  $g D l$ , at  $g$ , and  $l$ .
7. Draw  $l L$ , and  $g G$ , parallel to  $I M$ , cutting  $K F$ , and  $H E$ , at  $G$  and  $L$ ; make  $M K$  equal to  $M L$ , and  $M H$  equal to  $M G$ , then  $E H$ , and  $K L$ , will be the two axes required.





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## PROBLEM XIV.

An ellipsis  $A D B C$  being given, to draw a tangent through a given point  $H$  in the curve.

1. Find the foci  $F$  and  $G$ , join  $F H$  and  $G H$ .
2. Produce  $G H$  to  $I$ , upon  $H$ , with any radius, describe the arc  $K L I$ , cutting  $G I$ , and  $F H$ , at  $K$  and  $I$ .
3. Bisect the arc  $K L I$ , at  $L$ ; through  $L$  and  $H$  draw  $L H$ , it will be the tangent required.

## PROBLEM XV.

To draw two tangents to an ellipsis from a given point  $E$ , without it, having any two conjugate diameters  $A B$ , and  $C D$ , given, without drawing any part of the ellipsis.

1. Let the point  $E$  be in the diameter  $D C$  produced.
2. From the centre  $H$ , make  $H I$  equal to  $H C$ , and join  $I E$ .
3. Through  $C$ , draw  $C K$  parallel to  $I E$ , cutting  $H B$  in  $K$ .
4. Make  $H L$  equal to  $H K$ , through  $L$  draw  $F G$  parallel to  $A B$ , find the extreme points  $F$  and  $G$  of the ordinate  $F G$ ; by problem XI. From  $E$ , through the points  $F$  and  $G$ , draw  $E F$  and  $E G$ , they will be the tangents required.

If the point  $E$ , is in neither of the given diameters  $A B$ , or  $C D$ , when produced; draw a line from the given point  $E$ , through the centre; by problem XII. find a conjugate to that line, and the extremities of both, then the construction will be the same as in this.

PROBLEM

### PROBLEM XVI.

To describe an ellipsis similar to a given one  $A D B C$ , to any given length  $I K$ , or to a given width  $M L$ .

1. Let  $A B$ , and  $C D$ , be the two axes of the given ellipsis.
2. Through the points of contact  $A, D, B, C$ , complete the rectangle  $G E H F$ , draw the diagonals  $E F$ , and  $G H$ , they will pass through the centre at  $R$ .
3. Through  $I$ , and  $K$ , draw  $P N$ , and  $O Q$ , parallel to  $C D$ , cutting the diagonals  $E F$ , and  $G H$ , at  $P, N, Q, O$ .
4. Join  $P O$ , and  $N Q$ , cutting  $C D$  at  $L$ , and  $M$ , then  $I K$ , is the transverse,  $M L$  the conjugate axes of an ellipsis that will be similar to the given one  $A D B C$ , which may be described by some of the foregoing methods.

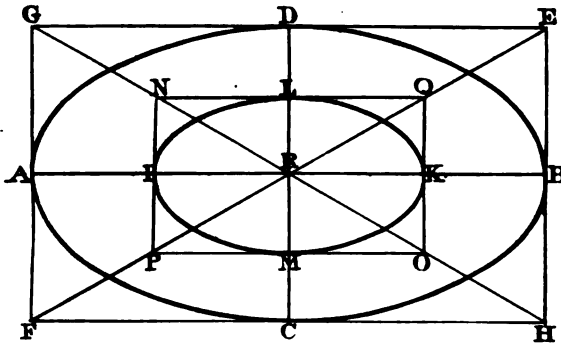
### PROBLEM XVII.

Given the rectangle  $A B C D$ , to circumscribe an ellipsis, which shall have its two axes in the same ratio as the sides of the rectangle.

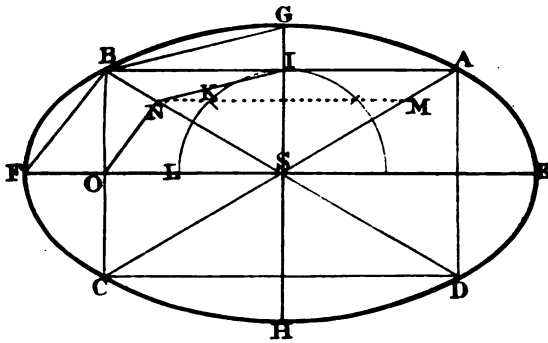
1. Draw the diagonals  $A C$ , and  $B D$ , cutting each other at  $S$ , the centre.
2. Through  $S$ , draw  $E F$ , and  $G H$ , parallel to  $A B$ , and  $A D$ .
3. Upon  $S$ , with a radius  $S I$ , equal to half  $A D$ , or  $B C$ , describe the quadrant  $I K L$ , cutting  $E F$ , at  $L$ .
4. Bisect the arc  $I K L$ , at  $K$ ; through  $K$ , draw  $M N$  parallel to  $E F$ , cutting the diagonal  $B D$ , at  $N$ .
5. Join  $I N$ ; through  $B$ , draw  $B G$ , parallel to it, cutting  $G H$  at  $G$ , and make  $S H$  equal to  $S G$ .
6. Join  $N O$ ; through  $B$ , draw  $B F$ , parallel to it, cutting  $E F$ , at  $F$ ; make  $S E$  equal to  $S F$ , then  $E F$ , and  $G H$ , are the two axes which may be described by some of the methods which are shewn in the foregoing problems.

PROBLEM

PROB. XVI.



PROB. XVII.

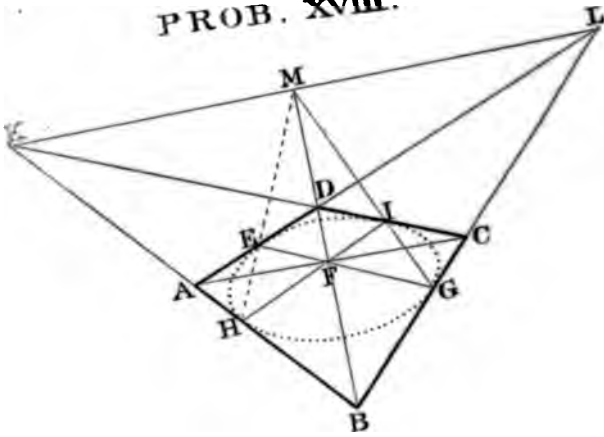




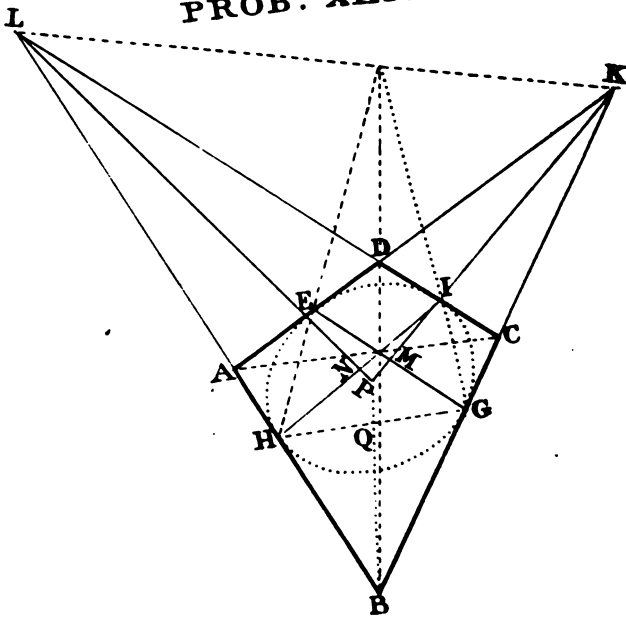
1

2

**PROB. XVIII.**



**PROB. XIX.**



## PROBLEM XVIII.

Given the trapezium  $A B C D$ , and a point  $E$ , in one of the sides, to find a point in each of the other sides, so that if an ellipsis was to be inscribed, it would touch the trapezium in these points.

1. Produce the sides of the trapezium, till they meet at  $K$  and  $L$ .
2. Draw the diagonals  $A C$ , and  $B D$ , cutting each other at  $F$ ; produce  $B D$ , till it cut  $K L$ , at  $M$ .
3. Through  $F$ , and the given point  $E$ , draw  $E G$ , cutting  $B C$  at  $G$ .
4. From  $M$ , through the points  $E$ , and  $G$ , draw  $M H$ , and  $M G$ , cutting the other two sides in the points  $I$  and  $H$ , then  $E, H, G, I$ , will be the four points required.

## PROBLEM XIX.

A trapezium  $A B C D$  being given, and a point  $E$ , in one of the sides, to find the centre of an ellipsis that may be described in the trapezium, and pass through the point of contact  $E$ , without drawing any part of the ellipsis.

1. Find the points of contact  $H, G, I, E$ , as in the last problem.
2. Join the points  $G$ , and  $E$ , by the right line  $G E$ ; bisect it in  $M$ , and from  $K$ , where the opposite sides  $A D$ , and  $B C$  meet, and through the point  $M$ , draw  $K M$  indefinitely.
3. Also join any other two points of contact, as  $H I$ ; bisect  $H I$ , at  $N$ , from  $L$ , where the opposite sides  $B A$ , and  $C D$  meet; draw  $L N$ , meeting  $K M$ , at  $P$ , then  $P$  will be the centre of the ellipsis required.

And in like manner if the points  $G$ , and  $H$ , were joined, and bisected at  $Q$ , and a line being drawn from  $B$ , where the opposite sides  $A B$ , and  $C D$  meet through  $Q$ , it would also meet in  $P$ , the centre, &c.

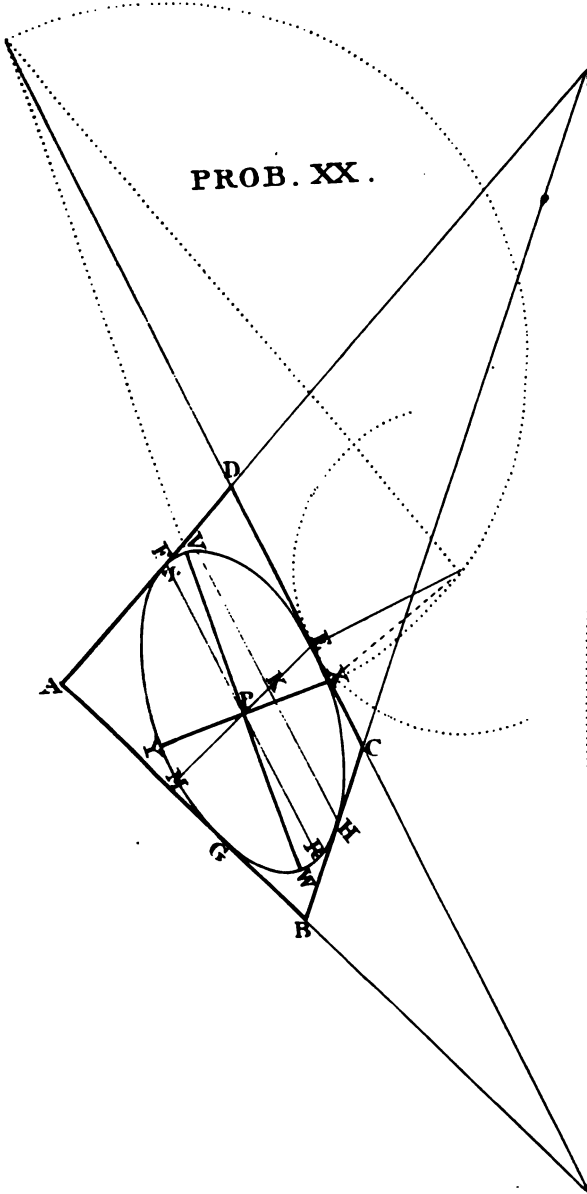
PROBLEM

## PROBLEM XX.

Given a trapezium  $A B C D$ , and a point  $E$ , in one of the sides, to find the two axes of an ellipsis that may be inscribed in the trapezium, and pass through the point  $E$ , without drawing any part of the ellipsis.

1. Find the opposite points of contact  $H$ ,  $E$ ,  $F$ ,  $G$ , by problem XVIII.
2. From thence, find the centre  $P$ , by the last problem.
3. From  $E$ , and through the centre  $P$ , draw  $E M$ , making  $P M$  equal to  $P E$ .
4. Through  $H$ , or any other point of contact, draw  $H K$ , parallel to  $D C$ , cutting  $E G$  at  $K$ ; then  $K H$  is an ordinate to the diameter  $E M$ .
5. Through  $P$ , the centre, draw  $P R$  parallel to  $H K$ .
6. Find the extremities  $R$  and  $S$ , of the diameter  $R S$ , by problem XI.
7. The conjugate diameters  $E M$ , and  $R S$ , being now found, then find the two axes  $V W$ , and  $X Y$ , by problem XIII.

PROB. XX.









# CONIC SECTIONS.

## OF THE PARABOLA.

### DEFINITION.

1. If a thread equal in length to  $BC$  be fixed at  $C$ , the end of a square,  $ABC$ , and the other end fixed at  $F$ ; and if the side  $AB$ , of the square be moved along the right line  $AD$ , and if the point  $E$  be always kept close to the edge  $BC$  of the square, keeping the string tight, the point or pin  $E$ , will describe a curve  $E G I H$ , called a *parabola*.

2. *Focus* is the fixed point  $F$ , about which the string revolves.

3. *Directrix* is the line  $AD$ , which the side of the square moves along.

4. *Axis* is the line  $L K$ , drawn through the focus  $F$ , perpendicular to the directrix.

5. *Vertex* is the point  $I$ , where the line  $L K$  cuts the curve.

6. *Latus rectum* or *parameter*, is the line  $G H$ , passing through the focus  $F$ , at right angles to the axis  $I K$ , and terminated by the curve.

7. *Diameter* is any line  $M N$  drawn parallel to the axis  $I K$ .

8. *Double ordinate* is a right line  $R S$ , drawn parallel to a tangent at  $M$ , the extreme of the diameter  $M N$ , terminated by the curve.

9. *Abcissa* is that part of a diameter contained between the curve and its ordinate, as  $M N$ .

## PROBLEM I.

To describe a parabola by finding points in the curve, the axis  $A B$ , or any diameter being given, and a double ordinate  $C D$ .

1. Through  $A$ , draw  $E F$  parallel to  $C D$ .
2. Through  $C$  and  $D$ , draw  $D F$  and  $C E$  parallel to  $A B$ , cutting  $E F$  at  $E$  and  $F$ .
3. Divide  $B C$  and  $B D$ , each into any number of equal parts, as four.
4. Likewise divide  $C E$  and  $D F$  into the same number of equal parts, viz. four.
5. Through the points 1, 2, 3, &c. in  $C D$ , draw the lines 1  $a$ , 2  $b$ , 3  $c$ , &c. parallel to  $C D$ .
6. Also through the points 1, 2, 3, in  $C E$  and  $D F$ , draw the lines 1  $A$ , 2  $A$ , 3  $A$ , cutting the parallel lines at the points  $a, b, c$ , then the points  $a, b, c$ , are in the curve of the parabola.

FIG. II. *Another Method.*

1. Join  $A C$  and  $A D$ ; from  $A$  make  $A E$  equal to  $B C$  or  $B D$ .
2. Through  $A$  and  $E$ , draw  $H I$ , and  $F G$ , parallel to  $C D$ , cutting  $A C$  and  $A D$  in the points  $F$  and  $G$ .
3. Through  $F$  and  $G$ , draw  $F H$  and  $G I$  parallel to  $A B$ , cutting  $H I$  at the points  $H$  and  $I$ .
4. From the points  $H$  and  $I$ , take any number of equal divisions on the lines  $H F$  and  $I G$ , from these points draw lines to  $A$ .
5. From  $B$ , set the same divisions towards  $C$  and  $D$ , draw the parallel lines 1  $a$ , 2  $b$ , 3  $c$ , &c. intersecting the former at the points  $a, b, c$ , they will be in the curve of the parabola.

CONIC

PROB. I.

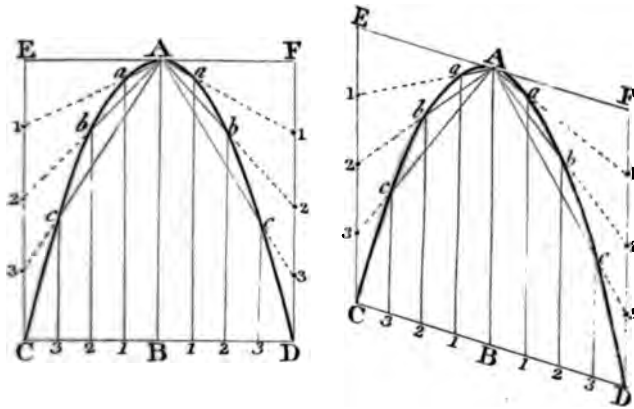
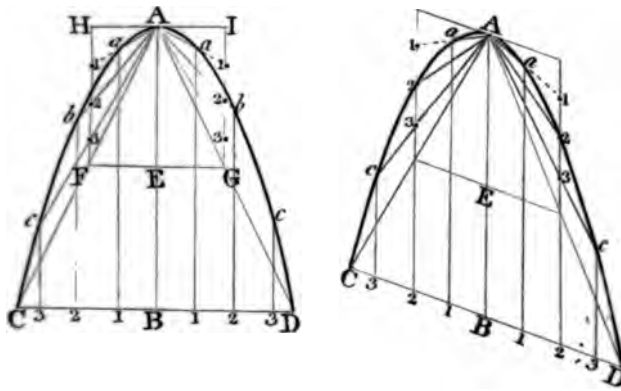


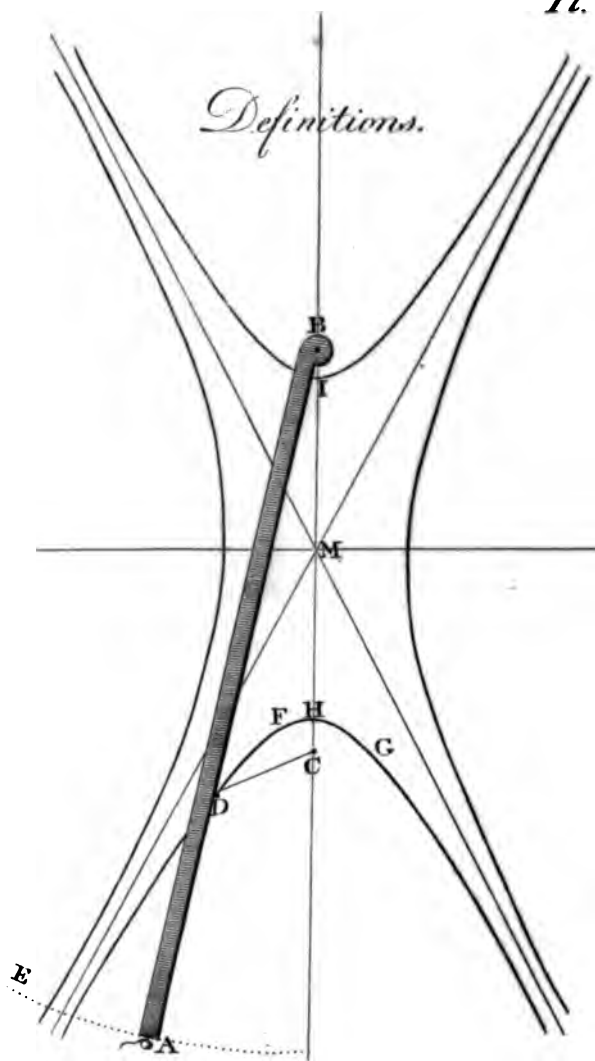
Fig. 2.







*Definitions.*



## CONIC SECTIONS.

### OF THE HYPERBOLA.

#### DEFINITION.

1. If B and C, are two fixed points, and a rule A B be made moveable about the point B, a string A D C being tied to the other end of the rule, and to the point C, and if the point A is moved round the centre B, towards E, the angle D of the string A D C, by keeping it always tight and close to the edge of the rule A B, will describe a curve D F H'G, called an *hyperbola*.

2. If the end of the rule at B was made moveable about the point C, the string being tied from the end of the rule A, to B, and a curve being described after the same manner, is called an *opposite hyperbola*.

3. *Foci* are the two points B, and C, about which the rule and string revolves.

4. *Transverse axis* is the line I H, terminated by the two curves passing through the foci if continued.

5. *Centre* is the point M, in the middle of the transverse axis I H.

6. *Conjugate axis* is the line  $N O$ , passing through the centre  $M$ , and terminated by a circle from  $H$ , whose radius is  $M C$ , at  $N$  and  $O$ .

7. *Diameter* is any line  $V W$ , drawn through the centre  $M$ , and terminated by the opposite curves.

8. *Conjugate diameter to another*, is a line drawn through the centre, parallel to a tangent with either of the curves, at the extreme of the other diameter, terminated by the curves.

9. *Abscissa* is when any diameter is continued within the curve, terminated by a double ordinate and the curve, then the part within is called the abscissa.

10. *Double ordinate* is a line drawn through any diameter, parallel to its conjugate, and terminated by the curve.

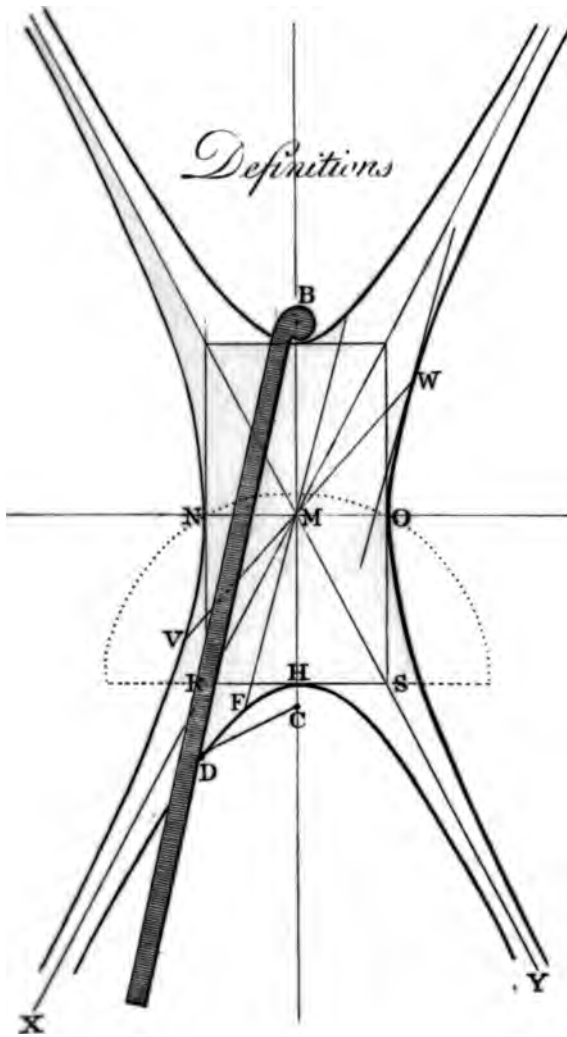
11. *Parameter, or latus rectum*, is a line drawn through the focus, perpendicular to the transverse axis, and terminated by the curve.

12. *Asymptotes* are two right lines drawn from the centre  $M$ , and the points  $R S$ , which is parallel to the conjugate axis  $N O$ , and drawn through the end of the transverse axis  $I H$ ;  $H R$ , and  $H S$  being equal to  $M N$  or  $M O$ , then  $M X$ , and  $M Y$ , are asymptotes.

13. *Equilateral or right angled hyperbola* is when its transverse and conjugate axes are equal.

## PROBLEM

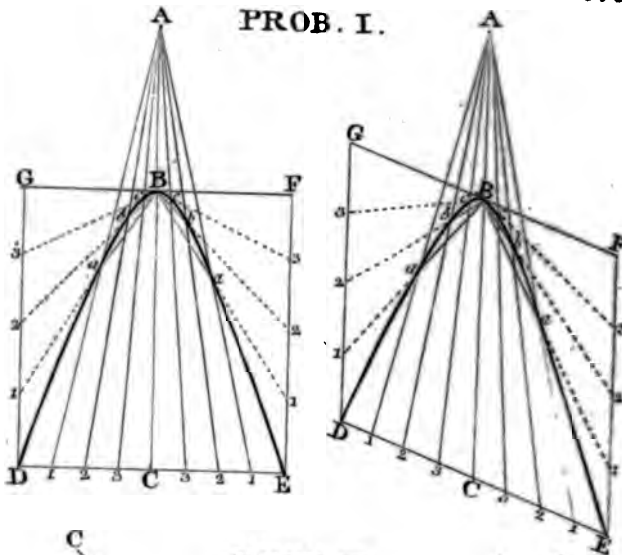
*Pl. 35.*



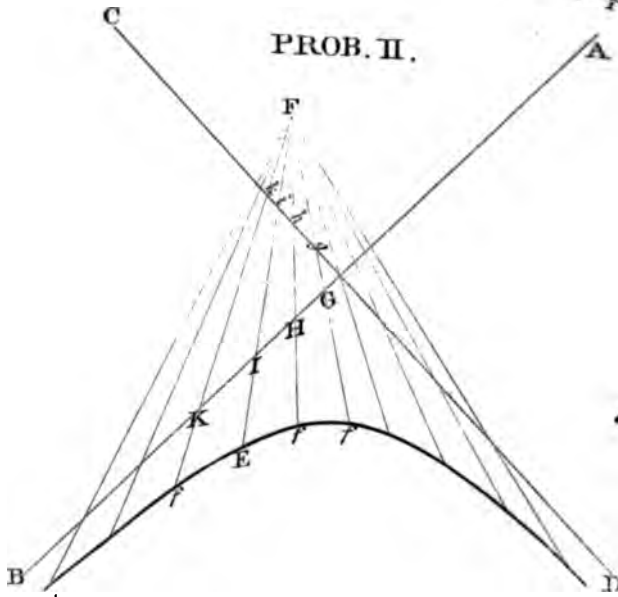




PROB. I.



PROB. II.



## PROBLEM I.

To describe an hyperbola by finding points in the curve, having the diameter, or axis  $A B$ , its abscissa  $B C$ , and double ordinate  $D E$ .

1. Through  $B$  draw  $G F$  parallel to  $D E$ ; from  $D$  and  $E$  draw  $D G$  and  $E F$  parallel to  $B C$ , cutting  $G F$  in  $F$  and  $G$ .

2. Divide  $C D$  and  $C E$ , each into any number of equal parts, as four: through the points of division 1, 2, 3, draw lines to  $A$ .

3. Likewise divide  $D G$  and  $E F$  into the same number of equal parts, viz. four; from the divisions on  $D G$  and  $E F$  draw lines to  $B$ , and a curve being drawn through the intersections at  $B a b c E$ , will be the hyperbola required.

## PROBLEM II.

Given the asymptotes  $A B$ ,  $C D$ , and a point  $E$  in the curve, to describe the hyperbola.

1. Through the given point  $E$  draw any right line  $E F$ , cutting the asymptotes in the points  $i$  and  $I$ .

2. Make  $i F$  equal to  $I E$ ; from  $F$  draw as many lines as you please, cutting the asymptotes in the points  $g, h, i, k, \&c.$  and  $G, H, I, K, \&c.$

3. Make  $G f, H f, K f, \&c.$  respectively equal  $g F, h F, k F, \&c.$  through the points  $f, f, f,$  describe a curve, and it is the hyperbola required.

In the same manner may the opposite hyperbola be described.

## PROBLEM

## PROBLEM III.

Given the two conjugate diameters  $AB$  and  $CD$ ,  
to find any number of points in the curve.

1. Through  $B$ , draw  $FG$  parallel to  $CD$ , make  $BF$ , and  $BG$ , equal to  $EC$ , or  $ED$ , from  $E$ , through  $F$ , and  $G$ , draw  $EH$ , and  $EI$ , the asymptotes.

2. From  $A$ , draw any lines  $AC$ ,  $AD$ ,  $AE$ , and  $AF$ , cutting the asymptotes at the points  $a$ ,  $a$ ,  $a$ , &c. and  $c$ ,  $d$ ,  $e$ ,  $f$ , &c. Make the distances  $aC$ ,  $aD$ ,  $aE$ ,  $aF$ , &c. equal to  $Ac$ ,  $Ad$ ,  $Ae$ , and  $Af$ , &c. then the points  $C$ ,  $D$ ,  $E$ ,  $F$ , &c. will be in the curve.

---

FIG. II. *Another Method.*

1. From the centre  $E$ , draw  $EI$  perpendicular and equal to  $EA$  or  $EB$ .

2. Join  $BC$ , take any number of points  $F$ ,  $G$ ,  $H$ , in  $EB$ , and draw  $Ff$ ,  $Gg$ ,  $Hh$ , parallel to  $BC$ , cutting  $EC$ , at  $f$ ,  $g$ ,  $h$ .

3. Through  $f$ ,  $g$ ,  $h$ ,  $C$ , draw  $fk$ ,  $gl$ ,  $hm$ ,  $in$ , &c.; take the distance  $FI$ , and make  $fk$ , equal to it; then take  $GI$ , and make  $gl$ , equal to it; in the same manner find the points  $mn$ . And if  $EB$  and  $EC$  are produced indefinitely beyond  $B$ , and  $C$ , and lines be drawn parallel to  $BB$ , as before, any number of points beyond will be found in the same manner.

---

PROBLEM IV.

Given the asymptotes, and a point in the curve,  
to find two conjugate diameters.

1. From the point  $B$ , draw  $BHD$  parallel to the asymptote  $CG$ . Make  $HD$  equal to  $HB$ , draw  $DCE$ , making  $CE$  equal to  $CD$ . Make  $CA$  equal to  $CB$ , then  $DE$  is the conjugate to  $AB$ .

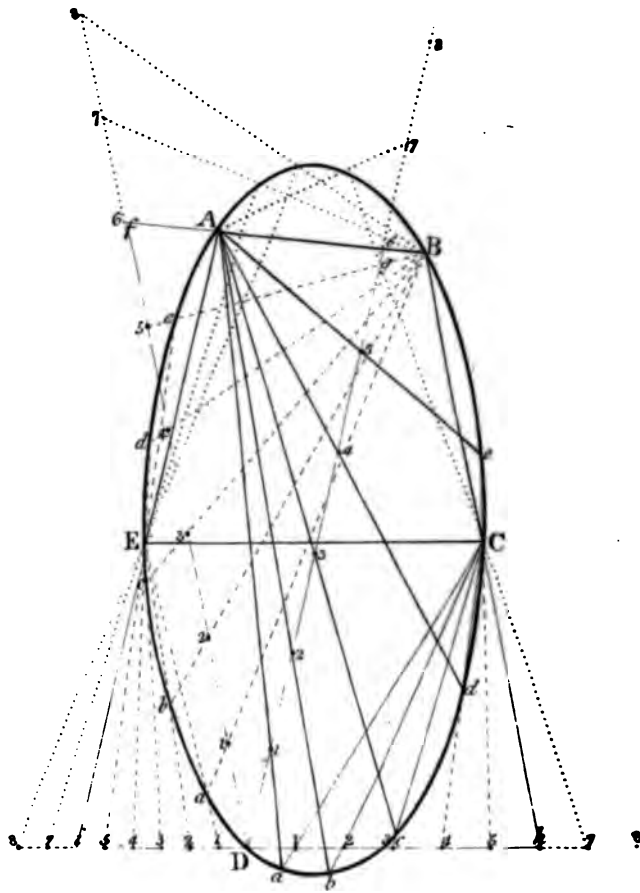
PROBLEM







*Pl. 38.*



## PROBLEM V.

To describe a conic section through five given points A, B, C, D, E, provided that all these points are joined by right lines, and that any exterior, or angle, formed by these lines, be less than two right angles.

1. Join any four points A, B, C, E, forming the quadrilateral A B C E.

2. Through the fifth point D, draw D *f*, and D *g*, parallel to A E, and B C, meeting A B produced both ways at the points *f* and *g*, if necessary.

3. Also through D, draw D *i*, parallel to E C, meeting B C, and A E, produced at the points *h*, and *i*.

4. Divide D *h*, D *i*, and D *f*, D *g*, into any number of equal parts, as six; likewise divide D F and D G, into the same, viz. six.

5. From the point *b*, and through the points 1, 2, 3, 4, 5, in D *i*, draw the lines 1 E, 2 E, 3 E, 4 E, 5 E, cutting the lines B *a*, B *b*, B *c*, B *d*, B *e*, and B *f*, at the points *a*, *b*, *c*, *d*, *e*, drawn from B, through 1, 2, 3, 4, 5, in D F, which are all in the curve.

In the same manner, the points between B, and D, will be found, viz. by drawing lines from the points A, and C, through the lines D *g*, and D *h*.

And if the lines D *i*, and D *f*, are produced, and the equal parts 7, 8, 9, extended upon these lines, you would obtain as many points *g*, *h*, *i*, &c. between A and B.

PROBLEM

## PROBLEM VI.

To describe a conic section to touch a right line  $AB$ , in a given point  $C$ , to pass through three other points  $D$ ,  $E$ , and  $F$ .

1. Join  $DC$ ,  $EC$ , and  $DE$ ; through  $F$ , draw  $FA$ , and  $FB$ , parallel to  $FE$  and  $DC$ , cutting  $AB$ , at  $A$  and  $B$ .
2. Through  $F$ , draw  $GH$  parallel to  $DE$ , and produce the sides  $CD$ , and  $CE$ , to cut it at  $G$  and  $H$ .
3. Divide  $FG$ , and  $FH$ ,  $FA$ , and  $FB$ , each into any number of equal parts, as four.
4. From  $C$ , through 1, 2, 3, in  $FH$ , draw  $Ca$ ,  $Cb$ ,  $Cc$ , &c.
5. From  $E$ , through 1, 2, 3, in  $FH$ , draw  $1E$ ,  $2E$ ,  $3E$ , &c. cutting the former in the points  $a$ ,  $b$ ,  $c$ , which are in the curve.

In the same manner may points be found in the other side.

## PROBLEM VII.

To describe a conic section, to touch two right lines  $AB$ , and  $BC$ , in the points  $A$  and  $C$ , and to pass through a given point  $D$ .

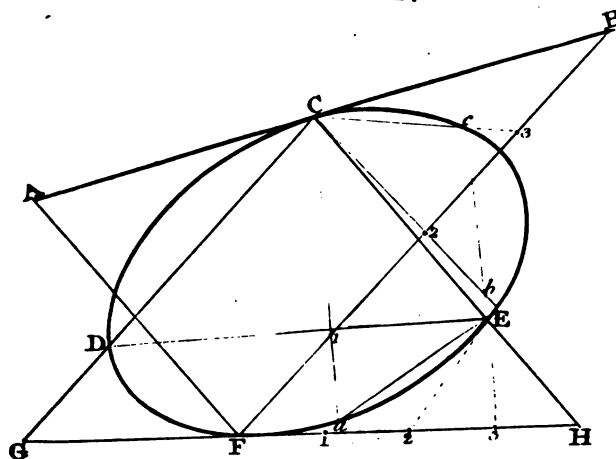
1. Join the points  $A$ , and  $C$ ; through  $D$ , draw  $DE$ , and  $DF$ , parallel to  $BA$ , and  $BC$ .
2. Through  $D$ , draw  $GH$ , parallel to  $AC$ , cutting  $BA$ , and  $BC$ , in  $G$  and  $H$ , and divide  $DG$ , and  $DH$ ,  $DE$ , and  $DF$ , each into the same number of equal parts.
3. From  $A$ , through the points 1, 2, 3, in  $DE$ , draw the lines  $Aa$ ,  $Ab$ ,  $Ac$ .
4. From  $C$ , through the points 1, 2, 3, in  $DH$ , draw  $1C$ ,  $2C$ ,  $3C$ , cutting the former in  $a$ ,  $b$ ,  $c$ , which are in the curve.

In the same manner may points be found between  $A$  and  $D$ .

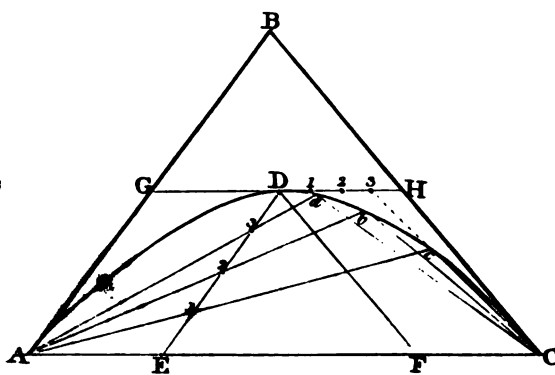
PROBLEM

*Pl. 39.*

PROB. VI.



PROB. VII.





## The Sections of Solids.

### OF A CYLINDER.

#### DEFINITIONS.

1. A cylinder is a solid generated by the revolution of a right angled parallelogram, or rectangle, about one of its sides, and consequently the ends of the cylinder are equal circles.

2. Axis is a right line passing from the centres of the two circles which form the ends of the cylinder.

3. If a cylinder is cut by a plane, parallel to a plane passing through its axis, it will be cut in two parts, which are called segments of the cylinder.

4. A segment of a cylinder is comprehended under three planes, and the curve surface of the cylinder; two of these are segments of circles: the other plane is a parallelogram, which is here for distinction's sake, called the plane of the segment, and the circular segments are called the ends of the cylinder.

5. The two sides of the parallelogram, which is parallel to the axis of the cylinder, is called the sides of the segment of the cylinder, and the other two sides of the parallelogram are chords to the ends of the cylinder.

6. If a cylinder, or segment of a cylinder, stands upon one of its ends, that end on which it stands is called the base.

7. If the segment of a cylinder is cut obliquely by a plane, the intersection of that plane, with the plane of the segment, is called the chord of the section.

8. The section of a cylinder cut by any plane inclined to its axis, is an ellipsis.

This is proved by the writers of conic sections.

## PROBLEM I.

To find the section of a semicylinder, cut by a plane at right angles to the plane  $A B F I$ , which passes through its axis, making a given angle  $E F B$ , with either of the sides  $B F$ .

1. Let  $A D B$  be the circle of the base, and  $C$  its centre.

2. Through the centre of the circle  $C$ , draw  $G D$ , parallel to  $F B$ , cutting the circle of the base in  $D$ , and  $E F$  at  $G$ ; from  $G$ , draw  $G H$  perpendicular to  $E F$ ; make  $G H$  equal to  $C D$ , then  $E F$  is the transverse axis, and  $G H$  the semiconjugate.

Or it may be described by ordinates, as in fig 2, taken from the base, and transferred to the section, as the figures direct.

In the same manner may any segment be found, viz. by drawing lines parallel to the sides of the plane of the segment, till it cut the chords of the section; from these points, draw perpendiculars to the chord, make their several lengths from the chord equal to those of the base corresponding to them; a curve line being drawn through these points, will be the true section of the segment required, as is plainly shewn by Fig. 3.

## PROBLEM

PROB. I.

Pl. 40.

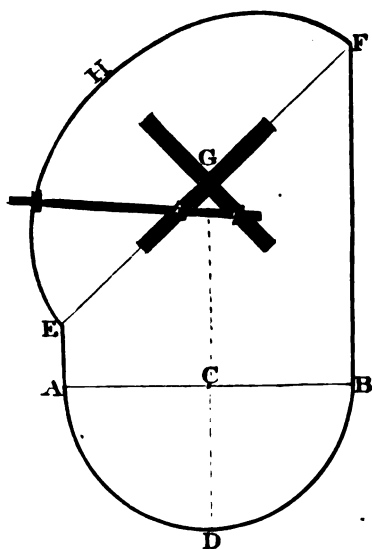


Fig. 2.

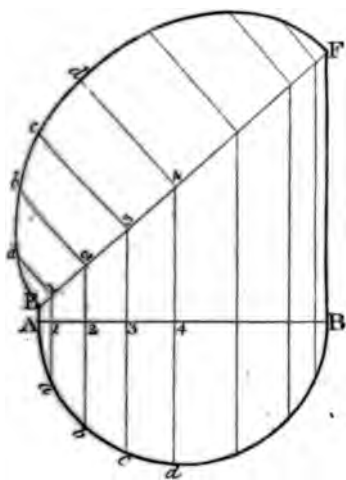
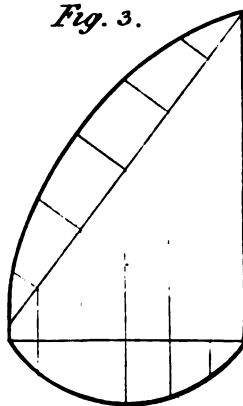
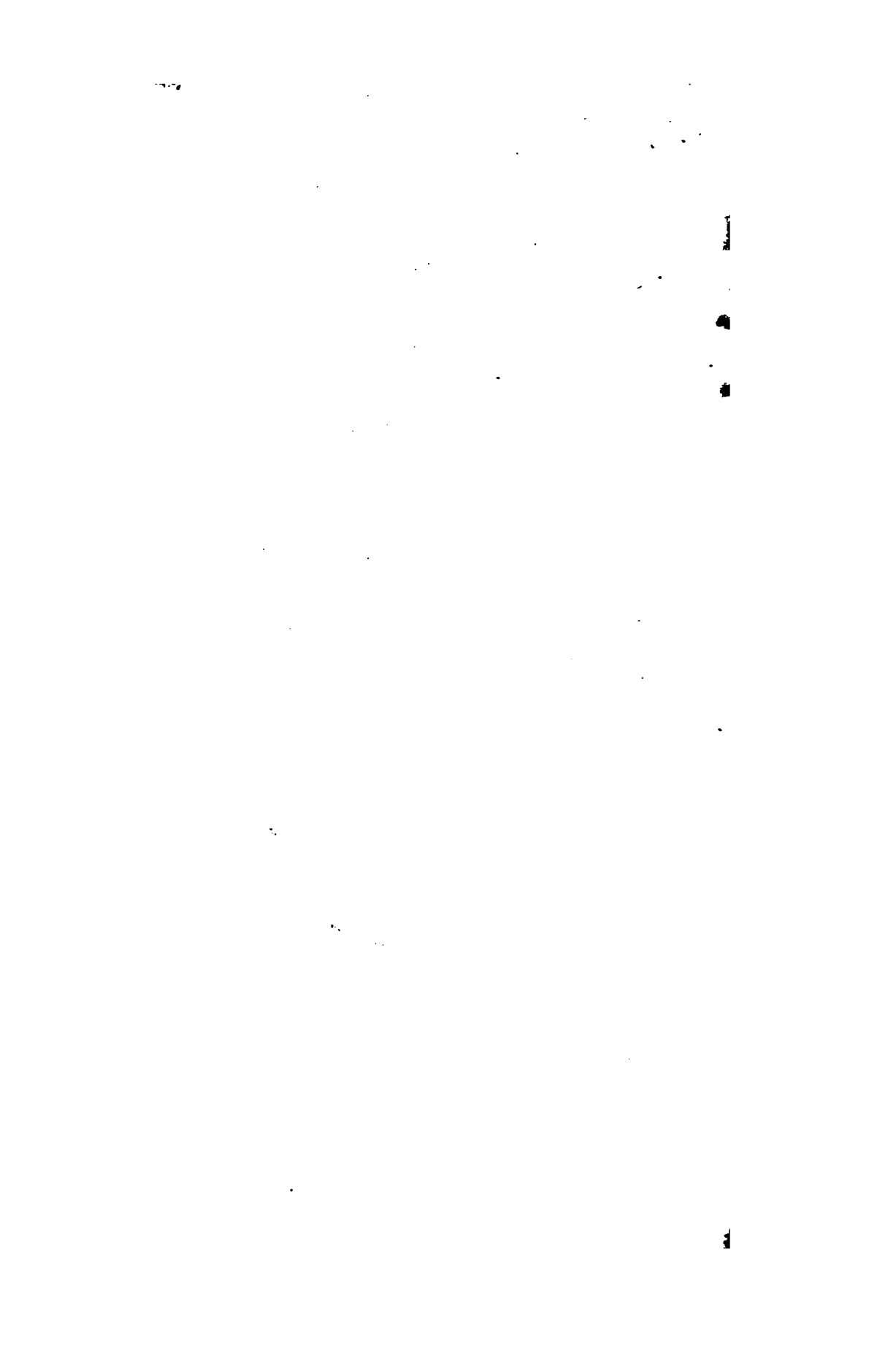


Fig. 3.

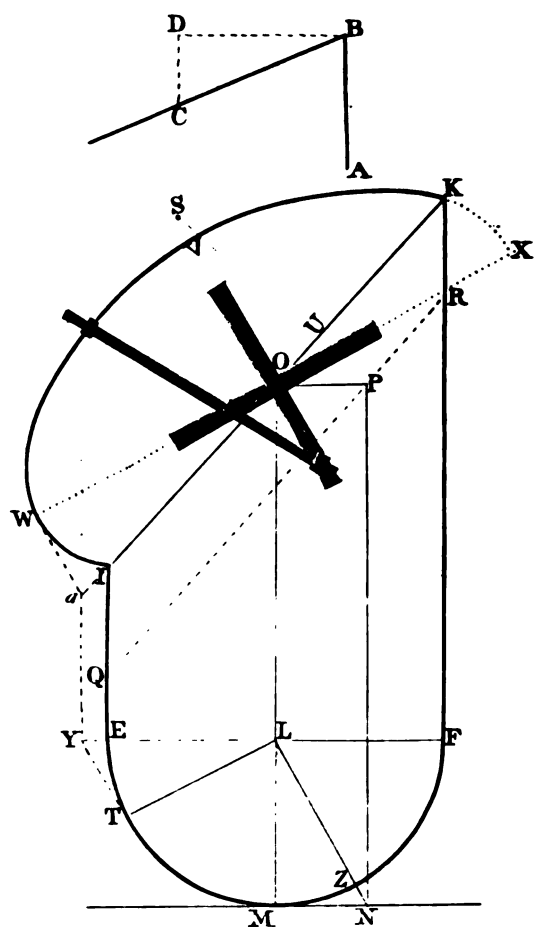






*Pl. 41.*

PROB. II.



## PROBLEM II.

To find the two axes of the section of a semi-cylinder cut by a plane, making a given angle  $A B C$ , with the plane  $E F G H$ , passing through its axis; also in a given direction  $K I$  with the side  $K F$ .

1. Let  $E M F$  be the circle of the base and  $L$  its centre.
2. From the angular point  $B$ , of the given angle  $A B C$ , draw  $B D$  perpendicular to  $B A$ , and equal to  $L E$  or  $L F$ , the radius of the base; draw  $D C$ , parallel to  $B A$ , cutting  $B C$  at  $C$ .
3. Draw  $Q R$ , at the distance  $D C$ , parallel to  $I K$ ; through the centre of the circle  $L$ , draw  $O M$  parallel to  $K F$ , cutting the circle of the base at  $M$ , and  $I K$ , at  $O$ ; through the point  $O$ , draw  $O P$  parallel to  $E F$ , cutting  $Q R$ , at  $P$ ; also through  $M$ , draw  $M N$  parallel to  $E F$ , from  $P$ , draw  $P S$  perpendicular to  $Q R$ , and  $P N$  parallel to  $O M$ , cutting  $M N$ , at  $N$ : join  $L N$ , cutting the circle at  $Z$ ; make  $U S$  equal to  $B C$ , and join  $O S$  upon  $O S$ ; from  $O$ , make  $O V$  equal to  $L Z$ , then  $O V$  is the semi-conjugate-axis.
4. Through  $O$ , draw  $W X$  perpendicular to  $O S$ ; draw  $L T$  perpendicular to  $L N$ , cutting the circle of the base at  $T$ ; from  $T$ , draw  $T Y$  parallel to  $L N$ , cutting the base  $F E$  produced at  $Y$ ; from  $Y$ , draw  $Y a$  parallel to  $O M$ , cutting  $K I$  produced at  $a$ ; from  $a$ , draw  $a W$  parallel to  $O S$ , cutting  $W X$ , at  $W$ ; make  $O X$  equal to  $O W$ , then  $W X$  is the transverse axis.

## PROBLEM III.

To find the section of a segment of a cylinder, by ordinates cut by a plane through a given line  $IK$ , in the plane of the segment, making a given angle  $ABC$ , at  $IK$ , with the plane  $EFKI$ .

1. Draw the tangent  $ZM$  parallel to  $EF$ , and draw  $OM$  parallel to  $KF$ , cutting the tangent at  $M$ , and  $IK$  at  $O$ ,

2. Take the distance  $LM$ , and make  $BD$ , perpendicular to the angular point  $B$ , of the given angle  $ABC$ , equal to it; proceed as in the last problem, find  $OV$ , and  $LZ$ .

3. Draw any number of lines  $a, b, c$ , &c. parallel to  $OL$ , cutting the lines  $IK$ , and  $EF$ , at the points  $a, b, c$ , &c.

4. From the points  $a, b, c$ , &c. in  $IK$ , draw lines  $a 1, b 2, c 3$ , &c. parallel to  $OV$ .

5. Through the points  $a, b, c$ , in  $EF$ , draw lines  $a 1, b 2, c 3$ , &c. parallel to  $LZ$ , cutting the arc line of the base, at  $1, 2, 3$ , &c.

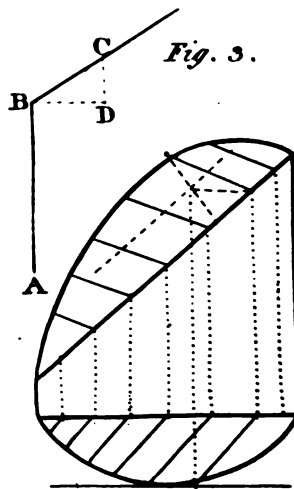
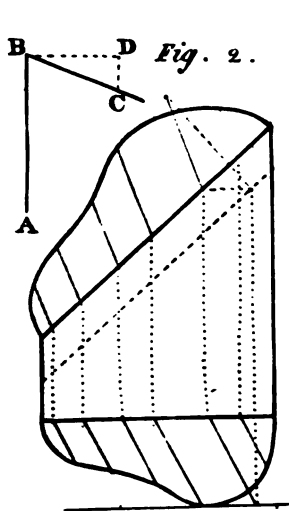
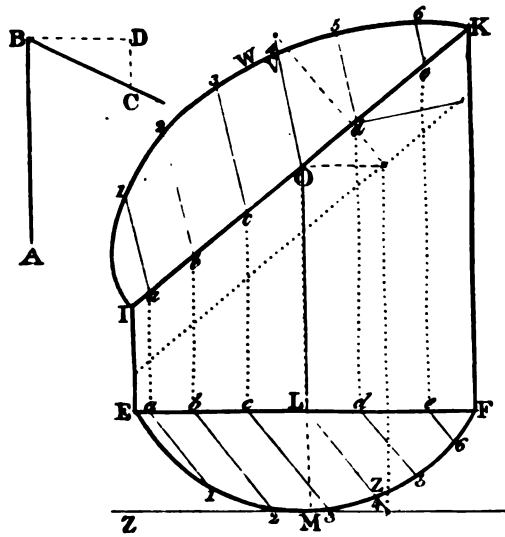
6. Make all the distances  $a 1, b 2, c 3$ , &c. from  $IK$ , equal to all their corresponding distances  $a 1, b 2, c 3$ , &c. on the base.

7. A curve line being traced through these points  $IWK$  will be the section required.

In the same manner the section of any irregular figure may be found, as is plainly shewn by Fig. 2.

Fig. 3. shows how to find the section when the angle  $ABC$  is oblique.

THE





## The Sections of Solids.

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### OF A CONE.

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#### DEFINITIONS.

1. A cone is a solid figure standing upon a circular base, diminishing to a point at the top, called its vertex, in such a manner, that if a straight line be applied from the vertex round the circle of the base, it shall coincide every where with the curve surface of the cone.

2. A right line passing through the cone, from the vertex to the centre of the circle at the base, is called the axis.

3. If a cone be cut by a plane, not parallel to its base, passing quite through the curve surface, the figure is an ellipsis.

4. If a cone be cut by a plane, parallel to a plane touching the curve surface, the section is a parabola.

5. If a cone be cut by a plane, parallel to any plane within the cone that passes through its vertex, then the figure is an hyperbola.

*These three last definitions are proved by the writers of conic sections.*

Note, the cone in the following problem, is supposed to be an upright one.

#### PROBLEM

## PROBLEM I.

To describe the conic sections from the cone.

*Note, A D N is a section of the plane, passing through its axis at right angles with the sections of the ellipsis, parabola, or hyperbola.*

*For the ellipsis.*

1. Let G H be its transverse-axis in the plane A D N; bisect it at K; through K, draw R Q parallel to A D.
2. Bisect Q R, at M, with the radius M R or M Q, describe the semicircle R P Q.
3. From K, draw K H, perpendicular to Q R, cutting the circle at H: then K H is the semiconjugate axis, from which the ellipsis may be described as at No. 1.

*For the parabola.*

1. Let S E be the axis of the parabola, parallel to the other side N D.
2. From E, draw E C at right angles to A D, the base, cutting the semicircle at C: then E C is an ordinate or half the base of the parabola, which may be described as at No. 2.

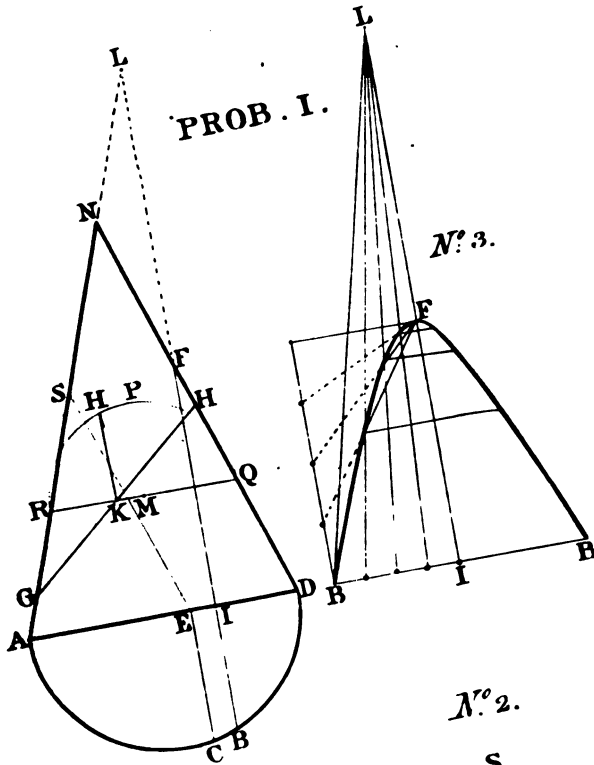
*For the hyperbola.*

1. Let I F be the height of the hyperbola, produce it till it cut the opposite side A N, produced at L, then F L is the transverse-axis.
2. From I, draw I B at right angles to A D: then I B is half the base, which may be described as at No. 3.

*Note, the letters are made to correspond at No. 1, 2, and 3, with those of the cone where they are taken from.*

THE

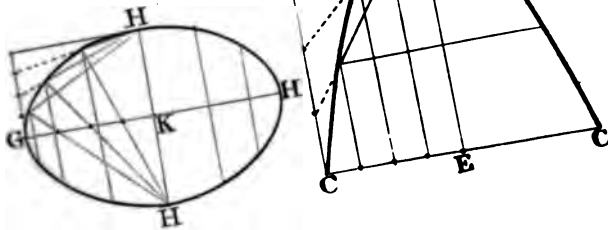
PROB. I.

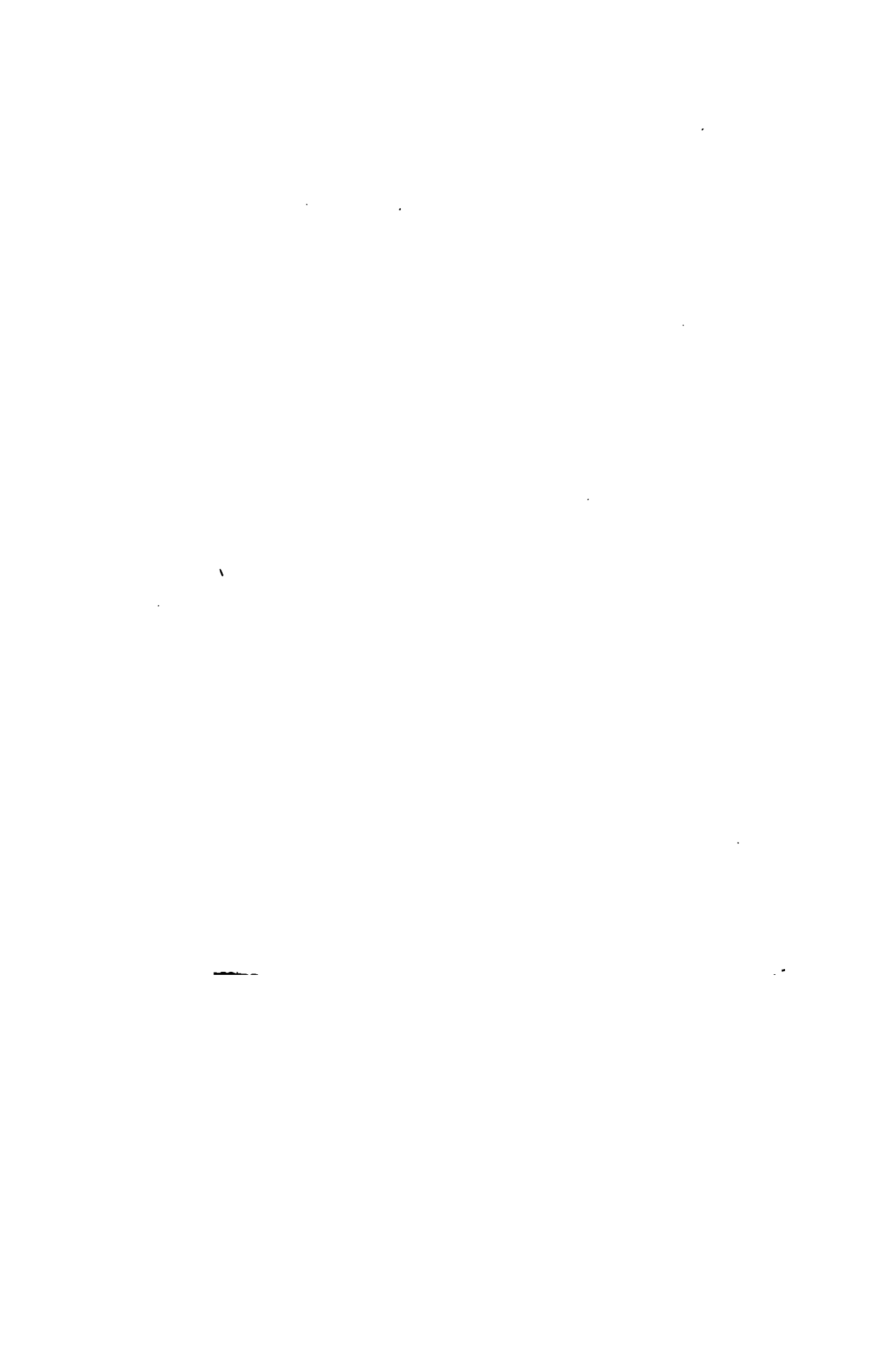


N° 3.

N° 2.

N° 1.





## The Sections of Solids.

### OF A GLOBE.

#### DEFINITIONS.

A globe is a solid figure, and may be supposed to be generated by the revolution of a semicircle about its diameter, which becomes the axis of the globe, and the centre of the semicircle is the centre of the globe.

*Corollary 1.* Hence all right lines drawn from the centre to the circumference of a globe are equal to another, for the semicircle touches the surface of the globe in every point as it revolves round.

*Corollary 2.* The section of a globe by a plane passing through its centre, is a semicircle, whose diameter is equal to the diameter of the generating semicircle.

*Corollary 3.* Every section of a globe cut by a plane, is a circle, for all the lines drawn from the centre to its surface, are equal, consequently the generating semicircle may revolve round any line as an axis, therefore every point in the semicircle will generate a circle.

*Corollary 4.* If a semiglobe is cut at right angles to the plane of its base, the section is a semicircle.

#### PROBLEM

## PROBLEM I.

To find the section of a semiglobe at right angles to the plane  $A B C$ , through its centre, and pass through the line  $A B$  in that plane.

Bisect  $A B$  in  $D$ ; on  $D$ , as a centre with the radius  $D A$ , or  $D B$ , describe the semicircle  $A E B$ , and it will be the section required.

## PROBLEM II.

Given two segments of circles  $A B C$ , and  $D E F$ , equal or unequal, having their two chords  $A C$ , and  $D F$  equal to each other, and the segment  $A B C$  being placed upon  $D F$ , so that  $A C$  shall coincide with  $D F$ , and the segment  $A B C$ , at right angles to  $D E F$ , to find the radius of a globe, so that the arc lines  $A B C$ , and  $D E F$ , shall be in its surface when the two segments are placed in the above position.

1. Make a rectangle  $A D F C$ , so that the opposite sides  $A C$ , and  $D F$ , will be the bases of the segments  $A B C$ , and  $D E F$ .

2. Find the centres  $G$  and  $H$ , of these segments.

3. Through  $H$ , draw  $I K$  parallel to  $D F$  and complete the semicircle  $I D E F K$ .

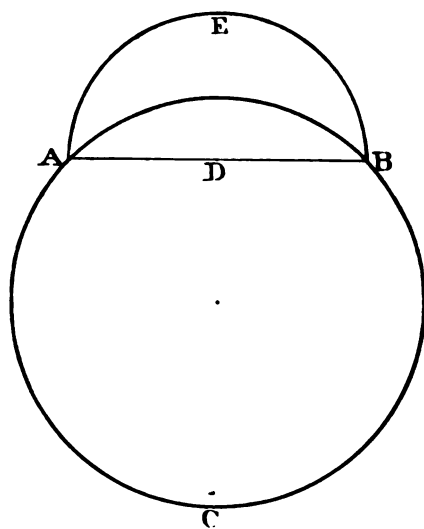
4. Through  $G$  or  $H$  draw  $H L$  parallel to  $G F$ , cutting  $A C$ , and  $I K$ , at  $L$  and  $H$ ; make  $H M$  equal to  $L G$ , join  $M K$  or  $M I$ , and it will be the radius required.

If upon  $M$ , as a centre, with the distance  $M I$ , or  $M K$ , a segment  $I N K$  is described, it will be part of the greatest circle that can be drawn in the globe.

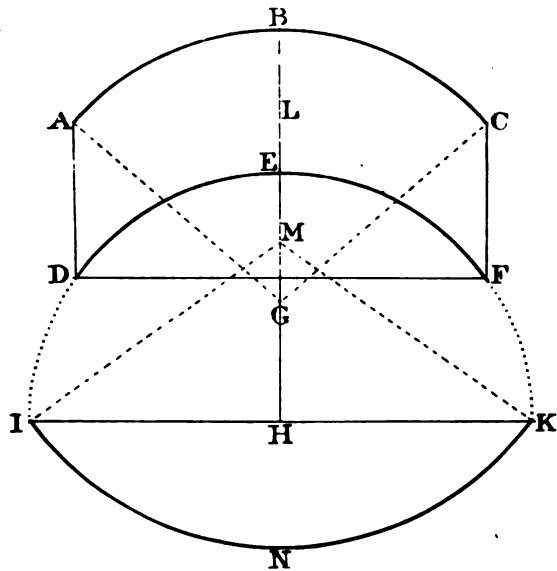
PROBLEM

PROB. I.

*Pl. 44.*



PROB. II.

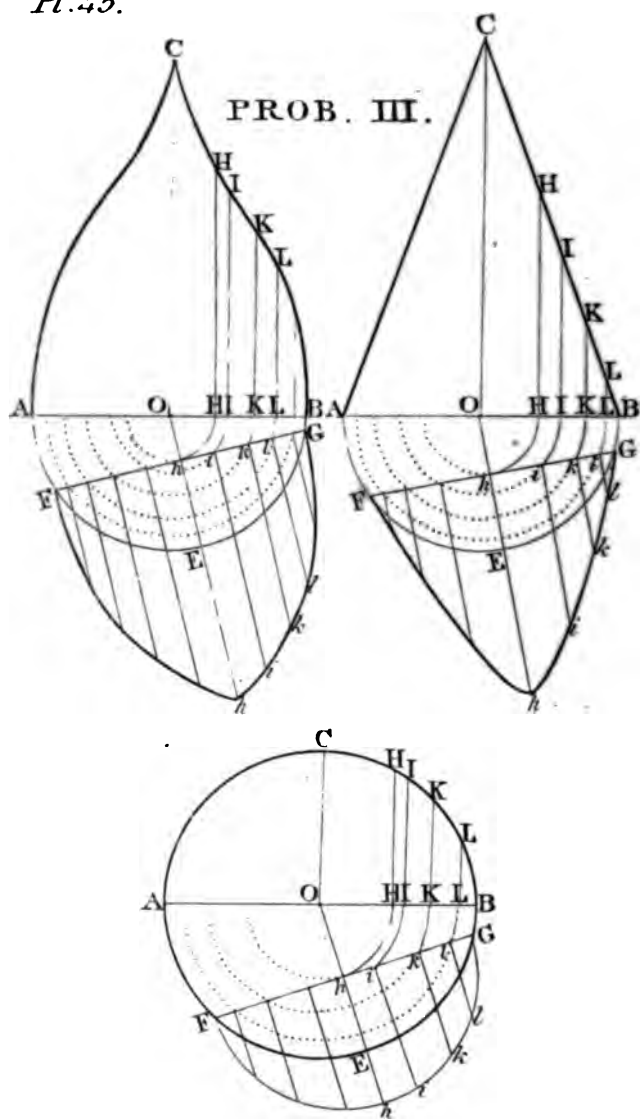






Pl. 45.

PROB. III.



## PROBLEM III.

A figure being generated by the revolution of a plain figure, having two perpendicular legs, and the other side being irregular, or straight, or a curve line of any kind; the figure being made to revolve about one of its perpendicular legs. To find the figure of the section cut any where across the base, and right angles to the plane of the base, having that section which passes through the axis given.

1. Let A F E G D be the circle of the base, and let the section required be cut across F G; also let A B C be a section of the solid passing through the axis.

2. From the centre O, draw the concentric circles H h I i, K k, L l, to cut A B in the points H, I, K, L: and F G, in the points h, i, k, l.

3. Erect perpendiculars to the lines A B and F G, both ways from these points, to cut A C in H, I, K, L.

4. Make the distances h h, i i, k k, l l, equal to their corresponding distances H H, I I, K K, L L; a curve being drawn through these points, it will be the section required.

If the given section is a triangle, the section is an upright hyperbola.

If the given section is a semicircle, the required section will also be a semicircle; these appear plain by the figures, and in this case there is no tracing required.

## The Sections of Solids.

### OF A SPHEROID.

#### DEFINITIONS.

1. A spheroid is a solid, generated by the rotation of a semiellipsis about the transverse or conjugate axis; and the centre of the ellipsis is the centre of the spheroid.
2. The line about which the ellipsis revolves, is called the axis.
3. If the spheroid is generated about the conjugate axis of the semiellipsis, then it is called a prolate spheroid.
4. If the spheroid is generated by the semiellipsis about the transverse axis, then it is called an oblong spheroid.

#### PROPOSITION I.

Every section of a spheroid is an ellipsis, except when it is perpendicular to that axis about which it is generated, in which case it is a circle.

#### PROPOSITION II.

All sections of a spheroid parallel to each other, are similar figures.

#### PROPOSITION III.

If a semispheroid is cut by a plane at right angles to the base,\* then the section is a semiellipsis, and the intersection with the base will be one of its axes; and if a line is drawn perpendicular from the middle of that intersection to the base of the spheroid, to cut its surface, that line will be half the other axis, whether transverse or conjugate.

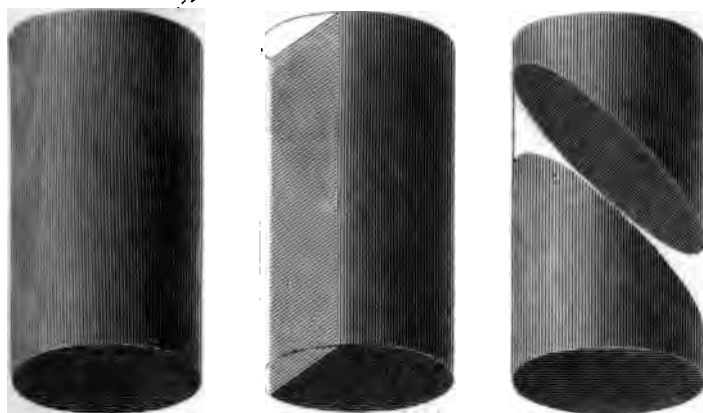
#### PROBLEM

\* It is here meant that the base is a section made by a plane, passing through the centre of the spheroid at right angles to the transverse or conjugate axis of the spheroid.

# SECTIONS of SOLIDS.

Pl. 46.

*A Cylinder and its Sections page 41.*

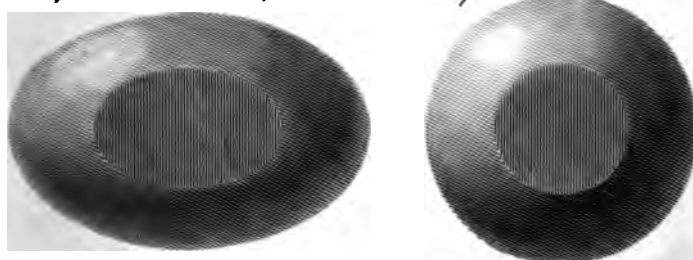


*A Cone and its Sections page 45.*

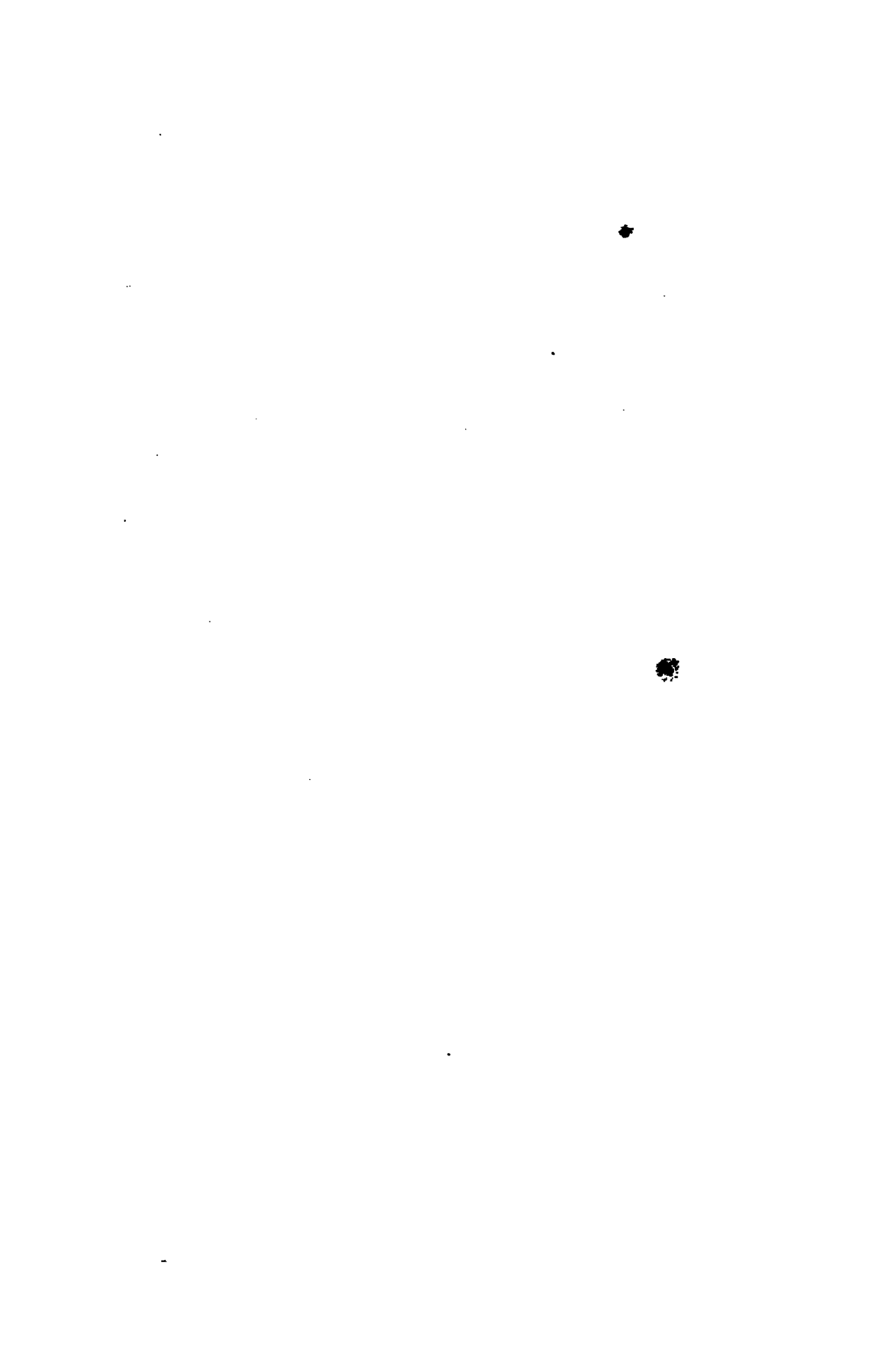


*A Spheroid &c. page 50*

*A Sphere or Globe page 47.*



*Engraved by W. J. Perry.*

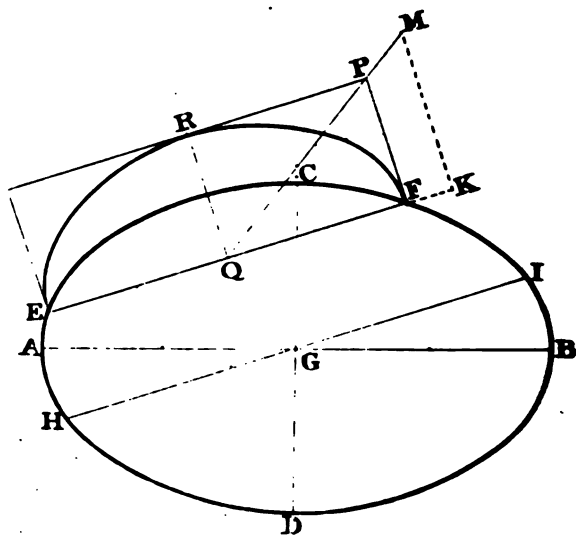




.

*Pl. 47.*

### PROB. I.



## PROBLEM I.

Given the base  $A D B C$ , which is a section through the longest axis of an oblong spheroid, to find the form of the section, by cutting the base through the line  $E F$ , at right angles to its plane.

1. Let  $A B$  be the transverse, and  $C D$  the conjugate axes of the base; through the centre  $G$  draw  $H I$  parallel to the given direction  $E F$ , cutting the ellipsis at the points  $H$  and  $I$ .

2. Produce  $E F$  towards  $K$ , make  $Q K$  equal to  $G H$  or  $G I$ ; and erect the perpendicular  $K M$ .

3. Make  $K M$  equal to  $C G$  or  $G D$ , and bisect  $E F$  at  $Q$ , and draw  $M Q$ . Erect the perpendicular  $F P$ , cutting  $M Q$  at the point  $P$ , and through  $Q$ , draw  $Q R$  parallel and equal to  $K M$ , then  $Q R$  will be the semiconjugate, and  $E F$ , the transverse axis of the section required, from which, the ellipsis may be described by any of the foregoing methods.

## PROBLEM II.

To find the length of any arc  $A B C$  of a circle mechanically, very near; or to transfer the same on the circumference of another circle  $F G H$  of a different radius, from a given point  $F$ .

1. Take your compass at any small opening, beginning at  $A$ , and take the equal parts 1, 2, 3, 4, 5, 6, on the arc  $A B C$ .

2. From  $D$ , lay the same number of equal parts on the right line  $D E$ , towards  $E$ , viz. 1, 2, 3, 4, 5, 6, and from the arc  $A B C$ , take the remaining part  $6 C$ , and place it on the right line  $D E$ , from 6 to  $E$ ; then will the length of the right line  $D E$  be nearly equal to the arc  $A B C$  stretched out.

In the same manner may  $A B C$  be transferred to the circle  $F G H$ , viz. by taking the divisions 1, 2, 3, 4, 5, 6, and beginning at  $F$ , with the same opening of your compass, setting off the divisions 1, 2, 3, 4, 5, 6, on the arc  $F G H$ , and transferring the part  $6 C$  to  $6 H$ , as before; then will the arc  $F G H$  be equal to the arc  $A B C$ .

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## Of a Cycloid or Epicycloid,

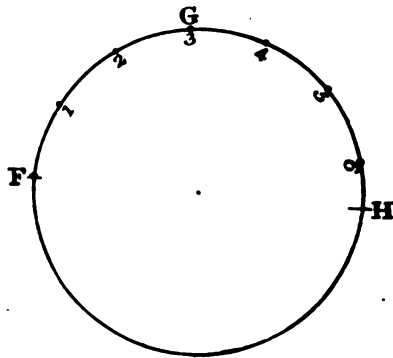
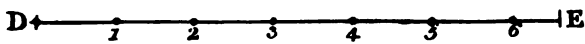
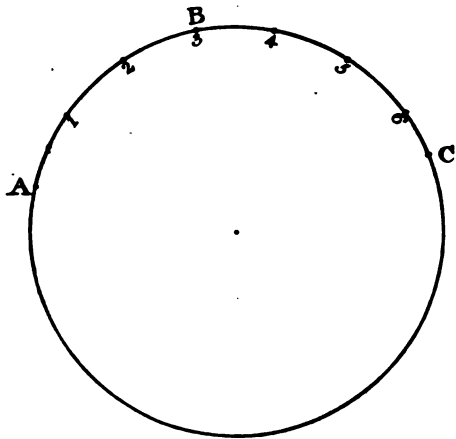
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### DEFINITION.

A cycloid or epicycloid, is a figure generated by a circle rolling along the straight edge of a ruler or another circle at rest, while a point in the circumference describes a figure on the plane called a cycloid or epicycloid.

### PROBLEM

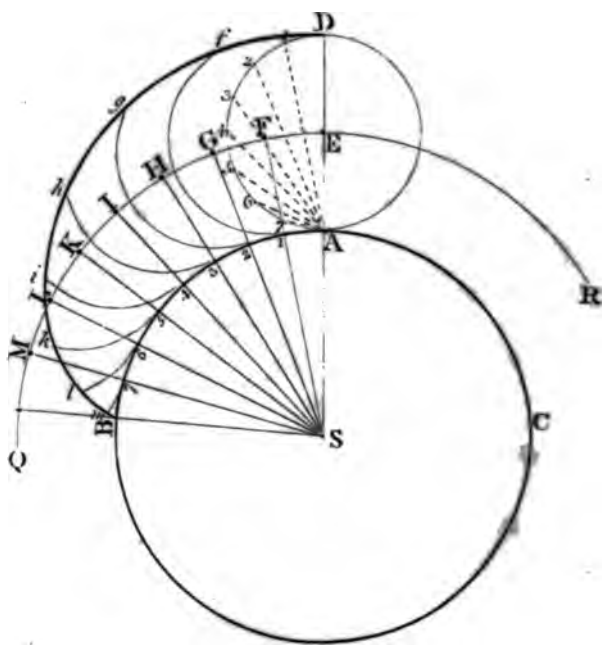
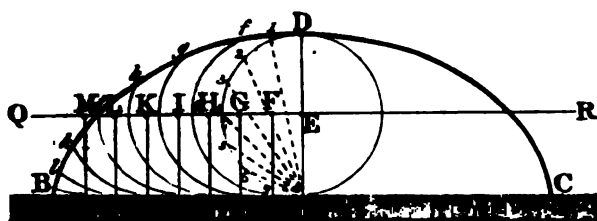
PROB. II. *Pl. 48.*







Pl. 49.



## PROBLEM I.

To describe a cycloid.

1. Let  $BC$  be the edge of a straight ruler: erect  $AD$  perpendicular to  $BC$  equal to the diameter of the generating circle; upon the diameter  $AD$ , describe a circle; through the centre at  $E$ , draw  $QR$  parallel to  $BC$ .

2. Divide the semicircumference  $D 1 2 3$ , &c. to  $A$ , into equal parts, and lay the same number of equal parts upon the right line  $AB$ , from  $A$  towards  $B$ ; from all the divisions on  $AC$ , erect perpendiculars, cutting  $QR$ , at the points  $F, G, H$ , &c.

3. With the radius  $ED$  or  $EA$ , on the points  $F, G, H$ , &c. as centres, describe arcs  $1 f, 2 g, 3 h$ , &c. take the chords  $A 1, A 2, A 3$ , &c. from the semicircle; make the distances  $1 f, 2 g, 3 h$ , &c. respectively to them, then these points will be in the curve of the cycloid.

## PROBLEM II.

To describe an epicycloid.

1. Let  $BAC$  be the edge of the circle round which the other circle is to turn; through the centre  $S$ , and the point  $A$ , in the circumference, draw the right line  $SD$ : make  $AD$  equal to the diameter of the generating circle.

2. Divide the circumference  $D, 1, 2, 3$ , &c. into equal parts and place them upon the arc  $AC$ , from  $A$  to  $1, 2, 3$ , &c. to  $8$ .

3. With the radius  $SE$ , on the centre  $S$ , describe the arc  $QR$ ; through the centre  $S$ , and the points  $1, 2, 3$ , &c. draw lines, cutting  $QR$  at  $F, G, H$ , &c. and proceed in every other respect, as in the cycloid, and you will get the curve.

END OF THE GEOMETRY.



# **PRACTICAL ARITHMETIC;**

CONTAINING SEVERAL

***USEFUL & NEW IMPROVEMENTS***

IN THE

**PRACTICE OF NUMBERS;**

**OR  
CONSISTING OF FOUR PARTS.**

**VIZ.**

<b>I. WHOLE NUMBERS,</b>	<b>:</b>	<b>III. DECIMAL FRACTIONS,</b>
	<b>:</b>	
<b>II. VULGAR FRACTIONS,</b>	<b>:</b>	<b>IV. DUODECIMALS.</b>

**WITH THEIR**

**APPLICATION TO MANY USEFUL EXAMPLES;**

*Worked out at full length, to illustrate the whole.*



\*~~~~~\*

# ARITHMETIC.

---

## DEFINITIONS.

I. **ARITHMETIC** is a science which explains the properties of numbers, shewing the method or art of computing by them.

II. *Unit* or *unity*, is only a single thing.

III. *Number* is a multitude of units.

IV. *Notation* teaches to express numbers by words or figures, or to read and write any sum or number.

## NOTATION.

The characters or figures, by which all numbers are expressed, are the following; 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 a cypher, sometimes called nought.

2 is 1 and 1 more.

3 is 2 and 1 more.

4 is 3 and 1 more.

5 is 4 and 1 more.

6 is 5 and 1 more.

7 is 6 and 1 more.

8 is 7 and 1 more.

9 is 8 and 1 more.

The number called 10 is 9 and 1 more.

Besides the above value of the figures, they have another, which depends upon the place they stand in, when joined together, as in the following table.

	Ninth place.	Eighth place.	Seventh place.	Sixth place.	Fifth place.	Fourth place.	Third place.	Second place.	First place.
Hundreds of Millions,	9	8	7	6	5	4	3	2	1
Tens of Millions,		9	8	7	6	5	4	3	2
Millions,			9	8	7	6	5	4	3
Hundreds of Thous.				9	8	7	6	5	4
Tens of Thous.					9	8	7	6	5
Thousands,						9	8	7	6
Hundreds,							9	8	7
Tens,								9	8
Units,									9

#### EXPLANATION.

I. All figures in the first perpendicular row, are units; so that 1 signifies one unit, 2 two units, 3 three units, &c.

II. Every unit in the second place, is ten times as great as an unit in the first place; so that 2 in the second place would signify two tens, and 3 three tens.

In general, every unit towards the left hand, is ten times as great as the next unit towards the right; so that an unit in the place of thousands, would be equal to ten hundred; every unit in the ten thousands in the fifth place, would be equal to ten thousand in the fourth place; every hundred thousand in the sixth place, would be equal to ten ten thousands in the fifth place, &c.

For

For the more easy reading of large numbers, they are divided into periods and half periods, each half period consisting of three places of figures; the name of the first half period being units; the name of the other half period thousands. Also the first figure in any half period is units; the second, tens; and the third, hundreds of that half period.

The following table exhibits a summary of the whole doctrine:

Quadrillions.	Trillions.	Billions.	Millions.	Units.
738,452.	436,136.	345,984.	412,624.	213,469.
th. un.	th. un.	th. un.	th. un.	th. un.

EXPLANATION.

Seven hundred thirty-eight thousand,  
four hundred and fifty-two - - - - - *Quadrillions.*  
Four hundred thirty-six thousand, one  
hundred and thirty-six - - - - - *Trillions.*  
Three hundred forty-five thousand,  
nine hundred and eighty-four - - - - - *Billions.*  
Four hundred twelve thousand, six  
hundred and twenty-four - - - - - *Millions.*  
Two hundred thirteen thousand, four  
hundred and sixty-nine - - - - - *Units.*

EXAMPLES.

*Write down in figures, thirty-five.* - - - - *Ans.* 35.  
*Write down in figures, one hundred and thirty-six* *Ans.* 136.  
*Write down in figures, one thousand three hundred and twenty.*  
*Ans.* 1320.  
*Write down in figures, one million and three thousand.*  
*Ans.* 1003000.  
*Write down in figures, three hundred thousand and one.*  
*Ans.* 300001.

From these examples it appears, though a cypher or 0 signifies nothing in itself, yet it serves to fill up the vacant places, and shows the true place of those figures which are towards the left hand of it.

## SIMPLE ADDITION.

### DEFINITION.

**A**DDITION is a rule which shows how to collect two or more numbers into one sum.

### NOTATION.

I. The character which denotes addition is marked  $+$ , which signifies the number that follows this mark, is to be added to the one which goes before it; thus,  $5+3$  signifies that the three is to be added to the five, and  $4+25+9$  signifies that 25 and 9 is to be added to the 4.

II. The sign  $=$  is the sign of equality; thus  $5+3=8$ .

### AXIOM.

The whole of any thing is equal to the sum of all its parts.

## PROBLEM I.

To add one or more numbers together.

I. Place the given numbers so that units will be under units, tens under tens, and hundreds under hundreds, &c.

II. Begin the place of units, reckon up all the figures in that place from bottom to top: take as many tens out of it as you can, set down the overplus, and carry the tens to the next row.

III. Add

III. Add the tens you carried from the last row, to the first figure in the next row of tens; to these, add all the remaining figures from the bottom to the top, and set down the overplus above ten, as before, and carry the tens to the next row of hundreds, and in this manner go through each, till you have completed the whole.

#### THE PROOF OF ADDITION.

Is to begin at the top, and add the numbers downwards; and if this sum agrees with the former, then the work is concluded to be right.

#### EXAMPLE.

*What is the sum of 785, 314, 409, 625, 483, 654, and 329?*

$$\begin{array}{r}
 785 \\
 314 \\
 409 \\
 625 \\
 483 \\
 654 \\
 329 \\
 \hline
 \text{Sum } 3599
 \end{array}$$

#### EXPLANATION.

Begin at the lower figure in the place of units, viz. 9, and say 9 and 4 is 13, and 3 is 16, and 5 is 21, and 9 is 30, and 4 is 34, and 5 is 39: set down 9, and carry 3 to the next row of tens.

Again say, 2 and 3 carried is 5, and 5 is 10, and 8 is 18, and 2 is 20, and 1 is 21, and 8 is 29: write 9 under, and carry 2 to the next row.

Lastly say, 3 and 2 carried is 5, and 6 is 11, and 4 is 15, and 6 is 21, and 4 is 25, and 3 is 28, and 7 is 35, which is set down in full, being the last row.

#### PROOF BY ADDING DOWNWARDS,

$5+4+9+5+3+4+9=39$ , set down 9 and carry 3.  
 Then  $3+8+1+2+8+5+2=29$ , set down 9 and carry 2.  
 Again  $2+7+3+4+6+4+3=35$ , which proves the whole work to be right.

EX-

Ex. II.	Ex. III.	Ex. IV.
3845	4305432	473216
4213	6953641	978543
6954	2649582	293847
3847	4638549	192337
7053	3756754	928374
<hr/>	<hr/>	<hr/>
25912	22303958	2866817
<hr/>	<hr/>	<hr/>

## EXAMPLE V.

$$409 + 3785 + 47 + 321 + 35 = 4597.$$

The reason for carrying one every time, is evident from the method of notation, and the total sum is equal to the sum of all the parts, which is evident from the axiom.



## SIMPLE SUBTRACTION.



## DEFINITION.

Subtraction is a rule for finding the difference between any two given numbers; the greater is called the minuend, and the lesser the subtrahend.

## NOTATION.

The character —, is the sign of subtraction; which signifies the number or numbers that have this mark placed before them, are to be subtracted or deducted from those numbers which are marked with a sign +\*.

Thus  $5 - 3 = 2$  and  $4 + 3 - 2 = 5$ , likewise  $5 + 3 + 6 - 3 - 4 = 7$ .

## AXIOMS.

\* The sign + is never used at any beginning number, but is always understood to be marked.

*AXIOMS.*

I. If equal numbers are added to unequal numbers, their difference are still the same as before; thus, the difference between 3 and 7 is 4; now let 10 be added to each, then the numbers will be increased to 13 and 17; but 17 taking 13 away, is also equal to 4.

II. The difference of two numbers added to the lesser, is equal to the greater; thus the difference between 3 and 5 is 2, which added to the least, viz. the 3, makes it also 5.

**PROBLEM II.**

**To subtract one number from another.**

I. Place the subtrahend under the minuend, so that units may be under units, tens under tens, &c.

II. Subtract the lower figure from the higher, if the upper figure is greatest, set the difference under; but if the lower figure is greater than the higher, add 10 to the higher, and set down the difference between the higher figure thus increased, and the lower.

III. If 10 was added to the higher figure in the first place, you must add 1 to the lower figure in the next place, and subtract it from the higher figure as in the first place; proceed throughout the whole in the same manner, setting each difference under their respective columns, and you will have the true difference of the whole.

The proof of subtraction is to add the difference to the subtrahend, if the work is right the sum will be equal to the minuend.

EX-

## EXAMPLE I.

*What is the difference between 736154, and 692839?*

Minuend	736154
Subtrahend	692839
	<hr/>
Remainder	43315
	<hr/>
Proof	736154
	<hr/>

## EXPLANATION.

As 9 cannot be taken from 4, 10 is added to the 4, which makes it 14; then 9 taken from 14, there remains 5, which is put under; because 10 was added to the upper figure in the last place, 1 must be added to the 3 in the next place, which makes it 4; then 4 taken from 5 there remains 1; write 1 under. Again, 8 from 1 I cannot, therefore I must add 10 to the 1, which makes it 11; then 8 from 11 there remains 3, which set down: and because 10 was borrowed to the upper figure in the last place, I must add 1 to the 2 in the next place, and proceed throughout the whole in the same manner.

## Ex. II.

From	78569663
Take	65862398
	<hr/>
Remainder	13207265
	<hr/>
Proof	78569663
	<hr/>

## Ex. III.

From	7963220
Take	2759668
	<hr/>
Remainder	5203552
	<hr/>

## EXAMPLE IV.

$$5630876 - 278566 = 5352310.$$

The

The reason of subtraction is evident; for when any figure in the greater number is less than its corresponding figure in the lesser or under number, the 10 which is added to the upper figure in that place, is equal to unity in the next place, by the method of notation: wherefore it follows, if 10 is added to the upper figure in any place, that 1 must be added to the lowest figure in the next place, therefore the two numbers, viz. the minuend and subtrahend, are equally augmented; then by the axiom their difference are still the same, and the whole difference being equal to the sum of the differences of all the similar parts, hence it follows, that the sum of the remainders of each correspondent place will be the true difference of the two given numbers.

---

## SIMPLE MULTIPLICATION.

---

### DEFINITIONS.

**M**ULTIPLICATION is a compendious method of adding any given number a certain number of times to itself; hence multiplication shows how to find a number to contain another any given number of times.

II. The number repeated is called the multiplicand.

III. The number of times by which the multiplicand is repeated, is called the multiplier.

IV. The multiplier and multiplicand, are each called factors.

V. The number arising, by repeating the multiplicand as often as there are units in the multiplier, is called the product.

*NOTATION.*

The character  $\times$  is the sign of multiplication, which being placed between two or more numbers, signifies that they are to be multiplied together; thus,  $3 \times 4 = 12$ , and  $4 \times 5 = 20$ ; likewise  $2 \times 3 \times 5 = 30$ , that is,  $2 \times 3 = 6$ , and  $6 \times 5 = 30$ .

*AXIOM.*

If the multiplier is separated into several parts, and each part being multiplied by the multiplicand, the sum of all the products will be equal to the multiplicand, taken as often as there are units in the multiplier, thus  $4 \times 3 = 12$ ; now let the multiplier, viz. the 3, be separated into the parts 2 and 1, then  $2 \times 4 = 8$ , and  $4 \times 1 = 4$ , but  $8 + 4 = 12$  as before.

For the ready multiplying of large numbers, the following table must be learned by heart, which contains the product of any two of the 9 digits.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

**PROBLEM**

### PROBLEM III.

To multiply one number by another.

I. Place the multiplier under the multiplicand, so that units may be under units, tens under tens, &c. and draw a line under them.

II. Begin at the units place, and multiply the multiplicand by each figure of the multiplier, observing to place the first figure of each product, under that figure of the product by which you multiply with.

III. Add the several products together, and the sum will be the product of the whole.

The most certain proof of multiplication, is to make the multiplier the multiplicand.

#### EXAMPLE I.

*What is the product of 364, multiplied by 25?*

Multiplicand	364	Proof.
Multiplier	25	25
	<hr/>	364
	1820	<hr/>
	728	100
	<hr/>	150
		75
Product	9100	<hr/>
	<hr/>	9100
		<hr/>

#### EXPLANATION.

Begin with the first figure of the multiplier, viz. the 5, and say  $4 \times 5 = 20$ : set down a cipher or 0 under, and carry 2 to the next place; then say,  $6 \times 5 = 30$ , and 2 carried makes 32: set down 2 and carry 3 to the next place; and say,  $3 \times 5 = 15$ , and 3 carried is 18, which is set down in full, being the last place.

Then begin with 2, the next figure, and say,  $4 \times 2 = 8$ , which is set under the 2; then say,  $6 \times 2 = 12$ : set down 2 and carry 1; and say,  $3 \times 2 = 6$ , and 1 carried is 7: which write down, then add the columns together, and the sum, viz. 9100, is the product.

Ex. II.

$$\begin{array}{r}
 \text{Multiply } 38543 \\
 \text{By } 4984 \\
 \hline
 8 \quad 154172 \\
 5+7 \quad 308344 \\
 8 \quad 346887 \\
 \quad 154172 \\
 \hline
 \text{Product } 192098312
 \end{array}$$

Ex. III.

$$\begin{array}{r}
 \text{Multiply } 300482 \\
 \text{By } 2364 \\
 \hline
 1201928 \\
 1802892 \\
 901446 \\
 600964 \\
 \hline
 \text{Product } 710339448
 \end{array}$$

There is another very expeditious method of proof, though not infallible, which will serve to confirm the truth of operations performed by multiplication, and is as follows :

First make a cross, add all the figures of the multiplicand together, omit the 9's out of the sum, and set down the excess on one end of the cross ; do the same with the multiplier, and set the excess on the opposite end of the cross ; then multiply the two excesses together, and leave out the 9's of the product, and if this last excess agrees with the excess above the 9's in the sum of the digits in the total product, then the work is concluded to be right.

Thus in *Example II.* the sum of the digits of the multiplicand, viz.  $3+8+5+4+3=23$ , the excess of the 9's in 23 is 5. Again, the sum of the digits in the multiplier is  $4+8+4=16$ , the excess above 9 is 7 ; then  $5 \times 7 = 35$ , and the excess of the 9's in the product is 8 : lastly, the excess of the sum of the digits in the total product is 8, which shows that the work is right.

To

**To contract the work in particular cases.**

When there are any ciphers in the middle of the multiplier, omit them; only observe to place the first figure of each product under that figure of the multiplier by which you multiply with: but if there are ciphers on the right hand of either the multiplicand or the multiplier, or in both, add the number of ciphers to the right hand product that there is to be found in each.

Ex. IV.	Ex. V.
379532	405832
305003	14800
<hr/>	<hr/>
1138596	3246656
1897660	1623328
1138596	405832
<hr/>	<hr/>
115758398596	6006313600
<hr/>	<hr/>

Ex. VI.	Ex. VII.
493754	3854800
30600	38000
<hr/>	<hr/>
2962524	308384
1481262	115644
<hr/>	<hr/>
15108872400	146482400000
<hr/>	<hr/>

**EXAMPLE VIII.**

$$4953698 \times 1000 = 4953698000.$$

**EXAMPLE IX.**

$$3750038000 \times 100000 = 375003800000000.$$

**EXAMPLE X.**

$$60000 \times 3000000 = 180000000000.$$

The

The reason of multiplying will appear evident; thus, if any two numbers are to be multiplied together, either of the two may be made the multiplier; thus,  $3 \times 4$  is the same as  $4 \times 3$ . To multiply any quantity, is to take the multiplicand as often as there are units in the multiplier; then 3 being multiplied by 4, is to take the 3 four times: again, when 3 is made the multiplier, it will be 3 times the units in 4; but there are 4 units in 4, and consequently there must be four 3's, that is, 4 times 3. The same reason will hold good for any two numbers whatever; and it will farther appear, when the multiplicand is considered as units, if the first figure of the multiplier is units, then the product of the multiplicand by the first figure, will also be units; and if the multiplicand be multiplied by any figure in the place of tens, then the product will also be tens: in like manner, when the multiplicand is multiplied by any figure of the multiplier, whether hundreds, thousands, &c. the product will be of the same kind with the multiplier, viz. hundreds, thousands, &c; but by the axiom, the total product is equal to the sum of all the products of the multiplicand, by each figure of the multiplier, which shews that the method is true.

SIMPLE

## SIMPLE DIVISION.

## DEFINITION.

**I. DIVISION** is a rule which shows how often one number is contained in another, or how often one number may be subtracted from another, which is the same thing; or how often one number will divide or measure another number; hence divisions is the reverse of multiplication.

**II.** The number to be divided is called a dividend.

**III.** The number you divide by is called the divisor.

**IV.** The number of times arising by dividing the dividend by the divisor, is called the quotient.

**V.** If the dividend contains the divisor any number of times, and a part be left that the divisor will not measure, that part is called a remainder.

## NOTATION.

The sign  $\div$  is the character which denotes division; thus  $12 \div 2 = 6$ , signifies that 12 is to be divided by 2, or that two is contained in 12 six times; likewise,  $36 \div 4 = 9$ , or it is often wrote in this manner,  $4)36(9$ , the middle number being the dividend; the number on the left hand, the divisor, and the number on the right hand, the quotient. Division is very often expressed in this manner;  $\frac{36}{4} = 9$ , that is, 36 divided by 4, is equal to 9: also,  $\frac{48}{12} = 4$ , which expressed, is 48 divided by 12, and is equal to 4; or the quotient arising by dividing 48 by 12 is 4. These two last are the most useful forms of denoting division.

## AXIOM.

*AXIOM.*

If any number is both multiplied and divided by the same number, it will be the same as if it had been neither multiplied nor divided, for multiplication shows how to find a number to contain another any given number of times; and division shows how often one number may be contained in another.

**PROBLEM IV.**

**To divide one number by another, or to find how often one number may be had in another.**

I. Write down the dividend, and draw a curve line on each side of it, to separate it from the divisor and quotient.

II. Write the divisor on the left, inquire how often it may be had in the same number on the left of the dividend, and write down the number of times on the right for the first quotient figure.

III. Multiply the divisor by the first quotient figure, and subtract the product from the dividend; but if the divisor cannot be had in the same figures as before, you must take one figure more in the dividend, than you have figures in the divisor, and proceed as before.

IV. Annex to the right of the remainder, if any, the next quotient figure, and inquire how often the divisor may be had in the remainder thus increased, and put the answer in the quotient as before; but if the divisor cannot be had in this remainder, you must bring as many more figures from the quotient as are requisite to contain the divisor, put as many ciphers in the quotient, excepting one, as you have  
got

got additional figures to the remainder, and write the number of times the divisor can be had in the remainder increased, on the right of the cyphers, and proceed as before, and the quotient is the answer.

The proof of division is by multiplication; if you multiply the divisor and the quotient together, and to the product add the remainder, the sum will be equal to the dividend.

EXAMPLE I.

*How often can 24 be contained in 36549?*

Dividend.		Proof.
Divisor 24	36549	1522
24	Answer	24
<hr/>		<hr/>
125		6088
120		3044
<hr/>		<hr/>
54		36528
48		+21 Remainder
<hr/>		<hr/>
69		36549—the divid.
48		<hr/>
<hr/>		
Remainder	21	

EXPLANATION.

First say, how often is 24 contained in 36? the answer is 1, which is set in the quotient; then say,  $24 \times 1 = 24$ , which put under the 36: then draw a line under, and put the difference 12 below, bring down the next quotient figure, viz. 5 to the 12, which will make 125; then inquire again how often the divisor can be had in 125, it will be found upon trial to be 5 times; then  $24 \times 5 = 120$ , which put under and subtract as before, and the remainder will be 5: to this, bring down the next quotient figure, viz. 4, and proceed as before, then you will find that 24 will be contained in 36549, 1522 times, and leave a remainder of 21.

$$\begin{array}{r}
 \text{Ex. II.} \\
 45923 \overline{)16761895(365} \\
 \underline{137769} \\
 298499 \\
 \underline{275538} \\
 229613 \\
 \underline{229613} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. III.} \\
 43042 \overline{)990139896(23004} \\
 \underline{86084} \\
 129299 \\
 \underline{129126} \\
 173896 \\
 \underline{172168} \\
 1728 \\
 \hline
 \end{array}$$

To contract the work in particular cases.

If there are any ciphers on the right hand of the divisor, omit them, and point off as many figures on the right hand of the quotient, as there are ciphers on the right of the divisor, and divide the remaining figures in the dividend by those of the divisor, and to the remainder, annex the figures cut off from the dividend, and you will have the whole remainder.

$$\begin{array}{r}
 \text{Ex. IV. contracted.} \\
 63 \overline{)003123729} \mid 38(49583 \\
 \underline{252}
 \end{array}$$

$$\begin{array}{r}
 603 \\
 \underline{567} \\
 367 \\
 \underline{315} \\
 522 \\
 \underline{504} \\
 189 \\
 \underline{189} \\
 \hline
 \text{Remainder} \quad 38
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. IV. at full length.} \\
 6300 \overline{)312372938(49583} \\
 \underline{25200}
 \end{array}$$

$$\begin{array}{r}
 60372 \\
 \underline{56700} \\
 36729 \\
 \underline{31500} \\
 52293 \\
 \underline{50400} \\
 18938 \\
 \underline{18900} \\
 \hline
 38
 \end{array}$$

From this example, it appears it would be evidently lost labour to divide by the ciphers, which in many cases would require more than double or treble the number of figures, than it would have by the contracted way.

When

When you have any single figure to divide by, you need not set down the operation at large, but multiply and subtract in your mind; write the quotient under the dividend, and the remainder at the end of the quotient, if any.

## EXAMPLE V.

[How often is 8 contained in 3290735?

$$\begin{array}{r}
 8 \overline{)3290735} \\
 \underline{411341} \quad \text{Rem.} \\
 \text{Answer} \quad 411341 \quad 7 \\
 \text{Quotient.}
 \end{array}$$

## EXPLANATION.

Eight cannot be contained in 3, but join the next figure to it, which will make it 32; then the 8's in 32, is four times, write 4 under: then the 8's out of the next figure, viz. 9, is once and one over, put 1 under the 9; then join the next figure 0, to the 1 that remained, it will make 10; then the 8's out of ten, is once and 2 over, write 1 under, and join the next figure to the 2, it will make 27; then the 8's in 27, three times and 3 over; proceed in this manner through the whole, and you will find your quotient to be 411341, and your remainder 7.

If you have a division with any number of ciphers on the right hand of a single digit, point off the number of ciphers in the divisor, and as many figures from the right hand of the dividend, as directed to contract the work in long division, and proceed as in the last example.

## EXAMPLE VI.

*What is the quotient of 573892648, divided by 4000?*

$$4 \mid 000)573892 \mid 648$$

Answer 143473    648 Remainder  
Quotient

## EXAMPLE VII.

*How often is 30000 contained in 375496583?*

$$3 \mid 0000)37549 \mid 6583$$

Answer 12516    16583 Rem.

When you divide by 10, 100, or 1000, &c. you need only point off so many figures from the right hand of the dividend, as you have got ciphers in your divisor, the remaining figures will be your quotient, and the figures pointed off will be the remainder; thus, 738495 divided by 100, will be 7385 | 49, the quotient being 7385, and the remainder 49. Also 495374 divided by 100000, is 4 | 95374.

## EXAMPLE VIII.

$$53796431 \div 10000 = 5379 \mid 6432.$$

## EXAMPLE IX.

$$400384000054 \div 1000000 = 400384 \mid 000054.$$

## EXAMPLE X.

$$456300000 \div 10000 = 45630 \mid 0000.$$

The

The reason of division will best appear by an example as under.

$$\begin{array}{r}
 36)163,24(400+50+3=453 \\
 \underline{144,00} \\
 192,4 \\
 \underline{180,0} \\
 12\ 4 \\
 \underline{10\ 8} \\
 \text{Rem.} \quad 1\ 6 \\
 \hline
 \text{Proof} \quad 1632\ 4
 \end{array}
 \qquad
 \begin{array}{r}
 14400 \\
 1800 \\
 108 \\
 16 \\
 \hline
 16324 \text{ Proof}
 \end{array}$$

From this example it appears, that the greatest number of hundreds that 36 can be contained in 16324, is 4, and the remainder is 1924; and the greatest number of tens that 36 can be contained in 1924, is 5, or 50, and the remainder is 124; and lastly, the greatest number of units that 36 can be contained in this last remainder, viz. 124, is 3, and the remainder 16, therefore the whole number of times that 36 can be contained in 16324, is  $400+50+3=453$  times; and by observing this work, it will be found to be the same as the general rule explains.

Division may be quickly proved by throwing out the 9's, as in multiplication; but as this method is not so certain a proof, it may be very easily proved by addition only, by adding the remainder to all the products of the divisor by the quotient figures, and if this sum is equal to the dividend, then the work is right.

Thus in the above example, begin at the right hand of the remainder, and say,  $6+8=14$ , set down 4 and carry 1; and say, 1 carried and 1 is 2, set down 2; and say,  $1+8+4=13$ , set down 3 and carry 1; and again say, 1 carried and  $1+4=6$ , set down 6; lastly, set down 1, and the whole is 16324, which agrees with the dividend.

Tables

## *Tables of Coins, Weights, and Measures.*



### MONEY.

		Marked
4 farthings	make 1 penny,	d.
12 pence	- 1 shilling,	s.
20 shillings	- 1 pound,	£

A farthing is always marked thus,  $\frac{1}{4}$ ; a halfpenny, or two farthings, thus,  $\frac{1}{2}$ ; and three farthings, thus,  $\frac{3}{4}$ .



### TROY WEIGHT.

By this weight are weighed gold, silver, amber, &c.

	Marked
24 grains make 1 penny weight . . .	dwts.
20 penny weights make 1 ounce . . .	oz.
12 ounces make 1 pound . . . . .	lb.

AVOIR-

## AVOIRDUPOISE WEIGHT.

By this weight are weighed, butter, cheese, flesh, grocery wares, and all goods that have waste.

	Marked
16 drachms make 1 oz. . . . .	dr.
16 ounces make 1 lb. . . . .	lb.
28 pounds make 1 quarter . . . . .	qr.
4 quarters make 1 hundred weight . . . . .	cwt.
20 hundred weight make 1 ton . . . . .	ton.

Note, that 14 ounces 11 penny weights, and 15 and a half grains troy, are equal to one pound Avoirdupoise.

## *A Table of Feet, Inches, &c.*

By this table, artificers in the building branch measure their work; but this sort of measure extends to length only.

	Marked
12 inches make 1 foot . . . . .	ft.
12 parts make 1 inch . . . . .	in.
12 seconds make 1 part . . . . .	p.
12 thirds make 1 second . . . . .	''
12 fourths make 1 third . . . . .	'''
&c. &c.	

## SQUARE MEASURE.

By this measure is measured all things that have length and breadth, such as carpenter's work, joiner's work, brick-layer's work, plasterer's work, &c.

144 inches make 1 foot.
9 feet make 1 yard.
100 feet make 1 square flooring.

COM-

## COMPOUND ADDITION.

---

### DEFINITIONS.

**C**OMPOUND Addition teaches to collect several numbers into one sum.

### PROBLEM V.

To add numbers of divers denominations together.

I. Place the numbers of the same denomination under each other, and draw a line under them.

II. Begin at the lowest denomination, and add up the figures in that place: find how many units of the next denomination is contained in that sum, by dividing it by so many as will make one of the next denomination.

III. Write the remainder or overplus underneath: add the quotient to the next place, and proceed through each column, as in the first place, writing the several remainders under their respective columns, till you get to the last place, which sum must be set down in full: by this means you will have the total sum of the whole.

EX-

Ex. I.	Ex. II.	Ex. III.
MONEY.	TROY WEIGHT.	AVOIRDUPOISE WT.
l. s. d.	lb. oz. dwt. grs.	ton. cwt. qr. lb. oz.
365 16 9	17 3 15 23	40 15 3 20 10
40 15 4½	14 6 3 20	1½ 3 2 0 11
306 12 1½	0 10 2 12	106 19 1 5 3
136 18 3	11 5 14 10	14 5 0 11 13
12 13 8½	12 11 1 0	5 10 2 27 12
410 11 4	10 4 6 11	112 18 3 25 15
<hr/> 1273 7 6½ <hr/>	<hr/> 67 5 4 4 <hr/>	<hr/> 294 13 2 8 0 <hr/>

EXPLANATION.

I. In example I. by adding up the farthings, you will find that their sum is 6, and because that four farthings is equal to one penny, therefore the 4's out of 6, is once and 2 over; set down 2, and carry 1 to the place of pence.

II. Add up all the pence together, as in simple addition, and their sum will be 30, and because that 12 pence is equal to 1 shilling, therefore divide 30 by 12, then the quotient is 2, and the remainder 6; set down 6, and carry 2, or two shillings, to the next place.

III. Proceed in the same manner with the shillings, and divide their sum by 20, because 20 shillings make one pound, then there is four 20's and 7 over, or 4 pounds 7 shillings, set down 7 and carry 4 to the place of pounds.

IV. Then add the pounds together, and set their sum under, then the whole sum will be 1273 pounds 7 shillings and 6 pence halfpenny.

In the same manner may any other kind of denominations be added together, by observing how many of any denomination make one of the next.

## EXAMPLE IV.

*A gentleman's house being finished, he orders the several artificers to send in their bills, including all the materials which they had found, which are as follows: how much has the whole house cost him in building?*

	£	s.	d.
Bricklayer's bill . . .	968	5	0
Carpenter's do. . . .	1786	16	4
Stone Mason's do. . .	650	14	5½
Plasterer's do. . . .	580	0	0
Painter's do. . . . .	460	16	9
Glazier's do. . . . .	198	15	6
Plumber's do. . . . .	168	14	6
Blacksmith's do. . . .	286	18	3
Paper-hanger's do. . .	250	12	6¾
Bell-hanger's do. . . .	100	0	0
	<hr/>		
	£ 5451	13	4 Ans.
	<hr/>		

## EXAMPLE V.

*How many shillings are there in a guinea, a pound, and a crown?*

21 shillings=a guinea.  
 20 shillings=a pound.  
 5 shillings=a crown.  
 —  
 Answer 46 shillings in the whole. ‘  
 —

## COMPOUND SUBTRACTION.

---

### DEFINITION.

**C**OMPOUND Subtraction is a rule for finding the difference between any two numbers of different denominations.

### PROBLEM VI.

**To subtract numbers of different denominations from each other.**

**I.** Place the greatest number above, and the lesser underneath, so that the same denominations may stand over each other, and draw a line under them.

**II.** Begin at the right-hand denomination, and subtract the lower place from the higher, and put the difference underneath.

**III.** If the lower denomination is greater than the higher, you must add as many to the higher place as will make one of the next denomination, then subtract the lower from the higher thus increased.

**IV.** Proceed to the next denomination, and add 1 to the lower place, if the upper place in the last denomination was increased, and subtract from the higher, as before, and proceed in this manner through the whole till it is finished, and the several remainders taken together will make the whole difference.

## EXAMPLE I.

*What is the difference between 98l. 17s. 6½d. and 48l. 12s. 9¼d.?*

	£	s.	d.
	98	17	6½
	48	12	9¼
	<hr/>		
Answer	50	4	8½
	<hr/>		

## EXPLANATION.

Because 3 farthings, or  $\frac{3}{4}$ , in the first place, cannot be taken from 2, or  $\frac{1}{2}$ , four farthings being equal to one of the next denomination, I therefore add 4 to the  $\frac{1}{2}$ , which makes it 6; then  $6-3=3$ : set down 3, and add 1 to the under place of the pence, viz. 9, which makes it 10; but as 10 is greater than the 6 above, I add 12, or 12 pence; which is equal to one shilling in the next denomination to the six, which makes 18; then  $18-10=8$ : set down 8, and add 1 to the 12 in the under place of the shillings, which makes it 13; then  $17-13=4$ ; lastly,  $98-48=50$ : write 50 under, then will 50l. 4s. 8½d. be the difference.

## Ex. II.

## TROY WEIGHT

	lb.	oz.	dwt.	gr.
From	18	10	15	8
Take	11	11	7	20
	<hr/>			
Diff.	6	11	7	12
	<hr/>			

## Ex. III.

## AVOIRDUPOISE WEIGHT.

	ton.	cwt.	qrs.	lb.	oz.	dwt.
From	56	19	3	10	15	14
Take	46	14	0	27	14	8
	<hr/>					
Diff.	10	5	2	11	1	6
	<hr/>					

EX-

EXAMPLE. IV.

*A Carpenter bought of a Timber Merchant, timber amounting to the value of 360l. 12s. 4d.; another quantity to the value of 286l. 14s. 2d.; at a third time, amounting to the value of 320l. 15s. 4d.: he then pays in part 480l. 16s. 8d.; how much more has he got to pay?*

£	s.	d.	
360	12	4	
286	14	2	
320	15	4	
<hr/>			
968	1	10	Timber Merchant's bill.
480	16	8	Paid by the Carpenter.
<hr/>			
Remains	487	5	2 Due to the Timber Merchant.
<hr/>			

EXAMPLE V.

*A piece of brick work was found to contain 8936 cubic feet of brick work, excepting three vaults, each containing 560 cubic feet: how many cubic feet is there in the brick work?*

560	
560	
560	
<hr/>	
1680	cubic feet in the vaults.
<hr/>	
8936	
—1680	
<hr/>	
Answer	7256 solid feet of brick work.
<hr/>	

## COMPOUND MULTIPLICATION.

### DEFINITION.

**C**OMPOUND Multiplication is a rule whereby we find the value of any given number, consisting of different denominations, or the amount, by repeating it any proposed number of times.

### PROBLEM VII.

To find the amount of any given number, repeated any proposed number of times.

Begin at the lower denomination, and multiply it by the number of times that the whole is to be repeated by; divide the product by as many as will make one of the next denomination: write the remainder underneath, and add the quotient to the next highest denomination, and proceed in this manner till all the denominations are multiplied.

### EXAMPLE I.

*What is the product of 35l. 16s. 4½d. repeated 6 times?*

$$\begin{array}{r}
 35 \text{ } 16 \text{ } 4\frac{1}{2} \\
 \times 6 \\
 \hline
 \text{£ } 214 \text{ } 18 \text{ } 3
 \end{array}$$

### EXPLANATION.

$6 \times 2 = 12$ , and  $12 \div 4 = 3$ : set down nothing, and carry 3 to the place of pence; then  $6 \times 4 + 3 = 27$ , and  $27 \div 12 = 2$ , and 3 over: set down 3, and carry 2 to the shillings; then  $6 \times 16 + 2 = 98$ , and  $98 \div 20 = 4$ , and 18 over: set down 18, and carry 4 to the pounds; lastly,  $6 \times 35 + 4 = 214$ .

Ex.

	Ex. II.					Ex. III.				
	FEET, INCHES, &c.					AVOIRDUPOISE WEIGHT.				
	ft.	in.	p.	''	'''	ton.	cwt.	qr.	lb.	oz.
Multiply	168	6	4	5	10	16	18	4	26	15
By	5					×4				
Product	842	7	10	5	2	67	16	3	23	12

To contract the work when you have a number above 9 to multiply by.

Find two or more digits, whose product may be equal to the given number; then multiply the given number by any of the digits, and the product by the other, &c. till you have multiplied by each digit, and the last product will be the answer: but if the product cannot be composed by 2 or more small numbers, find two or more numbers whose product is the nearest to the multiplier; then multiply by the parts, as before, add to the product the given number multiplied by the remainder, and the sum will be equal to the total product; but if the parts, when multiplied together, exceed the multiplier, subtract the difference multiplied by the given number, and the remainder is the answer.

#### EXAMPLE IV.

*What will 24 rods of brick work come to, at 7l. 16s. 5½d. per rod?*

$$\begin{array}{r}
 \text{£ } s. \text{ d.} \\
 7 \ 16 \ 5\frac{1}{2} \\
 \times 4 \\
 \hline
 31 \ 5 \ 10 \\
 \times 6 \\
 \hline
 \text{Answer } \text{£ } 187 \ 15 \ 0
 \end{array}$$

In this example, instead of multiplying by 24, I first multiply by 4, and that product by 6, because,  $4 \times 6 = 24$ .

EX.

## EXAMPLE V.

*What will 37 squares of flooring come to, at 4l. 18s. 6d. per square?*

£ s. d.	
4 18 6	
× 6	
29 11	
× 6	
177 6	Price of 36 squares.
+ 4 18 6	
182 4 6	Price of the whole.

In this example, because no two of the nine digits when multiplied, will make 37, but  $6 \times 6 = 36$ , which is the nearest of any 2 digits, only wanting one of 37, therefore I multiply the given price by 6, and that product again by 6, and add the given price to the product.

## EXAMPLE VI.

*What will the labour of 75 squares of flooring come to, at 16s. 4½d. per square?*

£ s. d.		£ s. d.	
0 16 4½		0 16 4½	
× 8		× 3	
6 11		2 9 1½	
+ 9			
58 19	Price of 72 squares.		
+ 2 9 1½	Price of 3 squares.		
61 8 1½	Price of the whole.		

COM.

## COMPOUND DIVISION.

---

### DEFINITION.

**C**OMPOUND Division is a rule, whereby any part of a given number, consisting of different denominations, may be found, or to find how often one number may be contained in another of different denominations.

### PROBLEM VIII.

To divide a number, consisting of different denominations, by a simple number.

I. Divide the highest denomination by the divisor, and put down the quotient.

II. Multiply the remainder, if any, by as many as will make one of the next denomination; to that, add the second denomination, if any, and divide by the same divisor as before, and proceed till you have got through each succeeding place.

The proof is by compound multiplication; if you multiply your quotient by the divisor, the product will be equal to the dividend.

## EXAMPLE I.

*Divide 126l. 16s. 6d. among 24 people, so that each may have an equal share.*

£	s.	d.	£	s.	d.	
24)	126	16	6(	5	5	8½ Answer.
	120			4		
	<hr/>			<hr/>		
	6			21	2	9
	×20			6		
	<hr/>			<hr/>		
	120			126	16	6 Proof.
	+16			<hr/>		
	<hr/>					
24)	136	5				
	120					
	<hr/>					
	16					
	×12					
	<hr/>					
	192					
	+6					
	<hr/>					
24)	198	8				
	192					
	<hr/>					
	6					
	×4					
	<hr/>					
24)	24	1				

When you have a single figure, or digit, to divide by, you need not set down the operation at large, but perform in your mind, as directed by the rule, setting each quotient under their respective denominations, and the several quotients together will be the answer.

**EX-**

EXAMPLE II.

*What is the sixth part of 236l. 15s. 6d.?*

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 6 \overline{) 236 \quad 15 \quad 6} \\
 \hline
 \text{Answer} \quad 39 \quad 9 \quad 3
 \end{array}$$

EXAMPLE III.

*If 8 feet of wood cost 1l. 15s. 2d. what will 1 foot cost?*

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 8 \overline{) 1 \quad 15 \quad 2} \\
 \hline
 \text{Answer} \quad 0 \quad 4 \quad 4\frac{1}{2}
 \end{array}$$

When you have a divisor consisting of the product of two or more digits, divide the dividend by either of them, as in the last examples, and that quotient by the other, &c. till you have divided all the parts, and the last quotient will be the answer.

EXAMPLE IV.

*If 36 feet of wood cost 3l. 16s. 8d. what will 1 foot cost?*

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 6 \overline{) 3 \quad 16 \quad 8} \\
 \hline
 6 \overline{) 0 \quad 12 \quad 9\frac{1}{2}} \\
 \hline
 \text{Answer} \quad 0 \quad 2 \quad 1\frac{1}{2}
 \end{array}$$

## REDUCTION.

---

### *DEFINITION.*

**R**EDUCTION is a rule which shows how to convert numbers from one denomination to another, still retaining the same value, and is either ascending or descending.

### PROBLEM IX.

To reduce a number of any kind into another, still retaining the same value, whether ascending or descending.

I. In reduction descending, multiply the highest denomination by as many as will make one of the next, and to the product add the next, if any, and proceed in the same manner from one denomination to another, until you arrive at that sought, observing to add to each product, as you proceed, those of the same with itself.

II. In reduction ascending, divide the given number by as many as make one of the next higher denomination, and that quotient by as many as make one of the next, &c. till you arrive at the denomination required; then the last quotient, with all the remainders annexed to it, will be the answer.

EX-

EXAMPLE I.

In 14l. 15s. 10d $\frac{1}{2}$ . how many shillings, pence, and farthings?

$$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 14 \quad 15 \quad 10\frac{1}{2} \\ \hline 20 \end{array}$$

$$\begin{array}{r} 295 \text{ Number of shillings.} \\ 12 \end{array}$$

$$\begin{array}{r} 3550 \text{ Number of pence.} \\ 4 \end{array}$$

Answer 14202 Number of farthings.

EXAMPLE II.

In 14202 farthings, how many pence, shillings, and pounds?  
farthings.

$$4)14202$$

$$12) 3550 \frac{1}{2}$$

$$2'0) 29'5 \ 10$$

Answer 14 15 10 $\frac{1}{2}$

EXAMPLE III.

In 18cwt. 3qrs. 26lb. 15oz. (*avoirdupois weight*), how many quarters, pounds, and ounces?

$$\begin{array}{r} \text{cwt.} \quad \text{qrs.} \quad \text{lb.} \quad \text{oz.} \\ 18 \quad 3 \quad 26 \quad 15 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 75 \text{ Number of quarters.} \\ 28 \end{array}$$

$$\begin{array}{r} 626 \\ 150 \end{array}$$

$$\begin{array}{r} 2126 \text{ Number of pounds.} \\ 16 \end{array}$$

$$\begin{array}{r} 12771 \\ 2126 \end{array}$$

Answer 34031 Number of ounces.

EX.

## EXAMPLE IV.

*In 34031 ounces (avoirdupois weight), how many pounds, quarters, and hundred weights?*

$$\begin{array}{r}
 \text{oz. } 28 \text{ } ^\circ 4 \\
 16 \overline{)34031(2126(75} \\
 \underline{32} \quad \underline{196} \quad \text{---} \\
 20 \quad 166 \\
 \underline{16} \quad \underline{140} \\
 43 \quad 26 \\
 \underline{32} \\
 111 \\
 \underline{96} \\
 15
 \end{array}
 \quad 18 \text{ } 3 \text{ } 26 \text{ } 15 \text{ Ans.}$$

## SIMPLE PROPORTION.

## DEFINITION.

**W**HEN four numbers are compared together, if the first be the same part, or parts, of the second, as the third is of the fourth, then the four numbers are said to be proportional, and are generally expressed, as the first is to the second, so is the third to the fourth: thus, 2, 6, 3, 9, are proportional numbers, for 2 is contained in 6 thrice, and 3 is contained in 9 thrice.

*Corollary.* Hence if four numbers are proportional, the quotient arising by dividing the second by the first, is equal to the quotient arising by dividing the fourth by the third.

II. In

II. In any proportion (2, 4, 6, 12), the two outside terms (2, 12) are called the extremes, and the two middle ones (4, 6), are called the means.

### NOTATION.

The signs of proportion are marked thus,  $\therefore$ ,  $\therefore\therefore$ , which being placed in the following manner,  $2 \therefore 8 \therefore\therefore 3 \therefore 12$ , signifies that the four terms, or numbers, are proportional; that is, 2 is to 8, as 3 is to 12.

### PROBLEM X.

Three numbers being given, to find a fourth proportional.

I. Place the term that is of the same kind with that which asks the question, first; that which is of the same kind with the term sought for, the second; and that which asks the question, the third: then say, if the first term given requires the second, what will the third require?

II. Multiply the second and third together, and divide the product by the first, and the quotient is the fourth term, or answer, which must be reduced as required.

*Note.* The first and third terms must be reduced to the same denomination.

To prove proportion, multiply the quotient by the first term, and the product will be equal to the dividend, or the two means multiplied together.

## EXAMPLE I.

*If 15 feet of wood cost 1l. 16s. 6d. what will 25 feet cost?*

## METHOD I.

$$\begin{array}{r}
 \text{£ } s. \text{ d.} \\
 1 \ 16 \ 6 \\
 20 \\
 \hline
 36 \\
 12 \\
 \hline
 \text{ft.} \quad \text{ft.} \\
 15 \ \ddagger \ 438 \ \ddagger \ 25 \\
 25 \\
 \hline
 2190 \\
 876 \\
 \hline
 \text{--- 12) } \\
 15)10950( \ 730 \ \text{Answer.} \\
 105 \quad \text{---} \\
 \text{--- } 2,0)6,0 \ 10 \quad \text{---} \\
 45 \quad \text{---} \\
 45 \ \text{£ } 3 \ 0 \ 10 \\
 \hline
 0 \\
 \hline
 \end{array}$$

METHOD II.  
by Compound Multiplication, &c.

$$\begin{array}{r}
 \text{£ } s. \text{ d.} \\
 15 \ \ddagger \ 1 \ 16 \ 6 \ \ddagger \ 25 \\
 \times 5 \\
 \hline
 9 \ 2 \ 6 \\
 \times 5 \\
 \hline
 3)45 \ 12 \ 6 \\
 \hline
 5)15 \ 4 \ 2 \\
 \hline
 \text{Answer } \text{£ } 3 \ 0 \ 10
 \end{array}$$

In Method I. the middle number being reduced to pence, the answer will also be in pence, and therefore it is reduced by reduction into pounds, shillings, and pence, as it will admit of them. In an example of this kind, where the terms can be resolved into simple parts, compound multiplication, &c. is the quickest way, as is shown in Method II.; but when the divisor cannot be separated into simple parts, I would prefer the common way, as is shown by Method I.

EX-

EXAMPLE II.

If 23 pound of lead cost 5s. 4½d. what will 1cwt. 3qrs. come to?

lb.	far.	lb.	s.	d.	cwt.	qrs.
23	258	196	5	4½	1	3
	196			12		4
	1548			64		7
	2322			4		28
	258					
	4)			258		56
23)50568	(2198		Farthings		14	
46						
	12) 549½				196lb.	
45						
23	2,0)4,5	9				
226	£ 2	5	9½			
207	Answer					
	198					
	184					
	14					

COMPOUND PROPORTION.

DEFINITION.

If any effect performed depend on two or more proportional causes, then it is called compound proportion.

## PROBLEM XI.

To resolve a question in compound proportion.

I. Say, as the product of all the given causes put for the first term, is to the given effect performed by these causes in the second term, so is the product of any other causes in the third term, to the effect performed by the last causes in the fourth term.

II. If the number sought be in the extremes, multiply all the means together for a dividend, and the extremes together for a divisor, and the quotient will be the answer.

III. But if the number sought is in the means, multiply the extremes together for a dividend, and the means together for a divisor, and the quotient will be the answer.

*Note.* It will be necessary to put some mark or letter where the term is wanting.

## EXAMPLE I.

*If 4 men, in 36 days, can lay 24 squares of flooring, how many squares will 12 men lay in 21 days?*

Here it is evident that the squares is the effect, or the work done.

$$\begin{array}{ccccccc} \text{causes} & & \text{eff.} & & \text{causes} & & \text{eff.} \\ 4 \times 36 & \vdots & 24 & \ddots & 12 \times 21 & \vdots & x \\ & & & & \times 12 & & \end{array}$$

---

252

$\times 24$

---

1008

504

---


$$4 \times 36 = 144 \quad 6048 \quad (42 \text{ squares the answer} = x.$$

576

---

288

288

In this Example, because the unknown terms fall in the extremes, I multiply the means together for a dividend, and the extremes for a divisor; then the work is performed as in simple proportion.

## EXAMPLE II.

*If 4 men, in 36 days, can lay 24 squares of flooring, how many men must be employed to complete 42 squares in 21 days?*

$$\begin{array}{rcl}
 \text{causes} & \text{eff.} & \text{causes} & \text{eff.} \\
 4 \times 36 & \div 24 & :: & x \times 21 \div 42 \\
 & & & \times 36 \\
 & & & \hline
 & & & 252 \\
 & & & 126 \\
 & & & \hline
 & & & 1512 \\
 & & & 4 \\
 & & & \hline
 24 \times 21 = 504 & & 6048 & (12 \text{ men} = x \\
 & & 504 & \\
 & & \hline
 & & 1008 & \\
 & & 1008 & \\
 & & \hline
 \end{array}$$

In this Example, because the unknown term is in the means, I therefore multiply the extremes together for a dividend, and divide by all the terms of the means multiplied together according to the Problem.

## EXAMPLE III.

*If 100l. in 12 months gain 5l. interest, what principal will gain the same sum in 8 months?*

$$\begin{array}{rcl}
 \text{causes} & \text{eff.} & \text{causes} & \text{eff.} \\
 100 \times 12 & \div 5 & :: & x \times 8 \div 5 \\
 12 & & & \\
 \hline
 8)1200 & & & \\
 \hline
 \text{£ 150 Principal.} & & & 
 \end{array}$$

In this Example, because the second and fourth terms are the same, it will be of no use to multiply and divide by the same number.

## EXAMPLE IV.

*There was a certain building completed by 150 workmen in 9 months, but by some accident it took fire, and was burnt down: how many men must be employed to complete a house twice as good as the former, so that it shall be finished in 12 months?*

$$\begin{array}{r}
 \begin{array}{cc} \text{causes} & \text{eff.} \end{array} & \begin{array}{cc} \text{causes} & \text{eff.} \end{array} \\
 150 \times 9 \div 1 & :: x \times 12 \div 2 \\
 9 & \\
 \hline
 1350 & \\
 2 & \\
 \hline
 12 \overline{) 2700} & \\
 \hline
 \text{Answer . 225 Workmen} = x. &
 \end{array}$$

## EXAMPLE V.

*If the carriage of 150 feet of wood, that weighs three stone a foot, for 40 miles, comes to 3l. how much will the carriage of 54 feet of stone, that weighs 8 stone a foot, cost for 25 miles?*

$$\begin{array}{r}
 \begin{array}{cc} \text{causes} & \text{eff.} \end{array} & \begin{array}{cc} \text{causes} & \text{eff.} \end{array} \\
 150 \times 3 \times 40 \div 3 & :: 54 \times 8 \times 25 \div x \\
 & 8 \\
 & \hline
 & 200 \\
 & 54 \\
 & \hline
 & 10800 \\
 & 3 \\
 & \hline
 150 \times 3 \times 40 = 18,000 & 32,400 (11. 16s. \text{ Ans.} \\
 & 18 \\
 & \hline
 & 14400 \\
 & 20 \\
 & \hline
 18 \overline{) 288,000} & (16 \\
 & 18 \\
 & \hline
 & 108 \\
 & 108 \\
 & \hline
 \end{array}$$

The

The reason of all operations in compound proportion, will appear in a similar manner to the following: thus, in Example V. the expence of the carriage will be in proportion to the number of feet, if the weight and distance carried are the same; and if the weight vary, and the number of feet and distance carried be the same, then will the expence be as the weight: but if the number of feet and the weight be the same, then the expence of the carriage will be as the number of miles; therefore, when neither of them are constant, the expence of the carriage will be as their product.

To contract an operation performed by compound proportion.

State the question as directed before, find a number that will divide any two terms, one of which must be in the extremes, and the other in the means; then scratch out the terms, and place their respective quotients above instead of them, and proceed in this manner as long as you can; then multiply and divide as directed by the problem, and you will have the answer.

#### EXAMPLE I.

*If 4 men in 36 days can lay 24 squares of flooring, how many squares will 12 men lay in 21 days?*

$$\begin{array}{ccccccc} & 3 & & 6 & & 1 & 7 \\ 4 \times 36 & : & 24 & :: & 12 \times 21 & : & x \end{array} \text{ then } 6 \times 7 = 42 \text{ sqrs. the ans.}$$

#### EXPLANATION.

It is easy to observe that 4 will divide 4 in the extremes, and also 24 in the means, therefore I place the quotient  
of

of  $24 \div 4 = 6$  above 24, and I scratch out the terms 4 and 24; I also find that 12 in the means will measure itself once, and 36 in the extremes 3 times; wherefore I scratch out the terms 12 and 36, and put the quotient 3 above 36; then I find that the 3 now found in the extremes will measure 21 in the means 7 times, which I scratch, as before, then there is only 6, and 7 remaining for the terms, which are both in the means; therefore I multiply the means for a dividend, and as there is no terms but units in the extremes to divide it, the product is the answer.

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## VULGAR FRACTIONS.

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### DEFINITIONS.

I. **A FRACTION** is some part or parts of an unit, or any whole number, and consists of two parts, the one called a numerator, and the other a denominator.

II. The denominator shows the number of parts the unit is divided into, and the numerator shows what number of these parts are to be taken.

*Corollary 1.* From this it is evident, if the number of parts expressed by the numerator, be equal to the number of parts in the denominator, then the fraction will be equal to an unit, and if the numerator is greater or less than the denominator, the value of the fraction will also be greater or less than an unit.

*Corollary 2.* Hence the value of any fraction is equal to the quotient arising, by dividing the numerator by its denominator.

denominator, together with the remaining parts, if any; therefore any vulgar fraction is properly expressed in this manner,  $\frac{a}{b}$ , the number above the line being the numerator, and the number below the denominator.

III. When the value of a fraction is less than an unit, such as  $\frac{1}{2}$ ,  $\frac{2}{3}$ , or  $\frac{3}{4}$ , it is called a proper fraction.

IV. When the value of a fraction is greater than an unit, such as these,  $\frac{3}{2}$ ,  $\frac{5}{3}$ ,  $\frac{7}{4}$ , &c. then the fraction is called improper.

V. If the numerator of an improper fraction be divided by its denominator, then the quotient, together with the fractional parts that is left, is called a mixed number; thus,  $\frac{7}{4} = 1\frac{3}{4}$ , or  $1\frac{3}{4}$ , is a mixed number.

VI. A compound fraction is the fraction of a fraction; thus,  $\frac{1}{2}$  of a  $\frac{3}{4}$  is a compound fraction.

## PROBLEM I.

To find the greatest common measure in two or more numbers.

I. If there are only two numbers, divide the greater by the less, and the divisor by the remainder, and proceed in this manner till nothing remains, then will the last divisor be the greatest common measure of the two numbers.

II. When there is more than two numbers, find the greatest common measure of any two of them, as before; also the greatest common measure of that common measure, and of the other numbers, and proceed in this manner through all the numbers to the last, then will the last common measure be the greatest that will measure all the given numbers.

EX-

## EXAMPLE.

*What is the greatest common measure of 108, 132, and 78?*

$  \begin{array}{r}  108 \overline{)132} 1 \\  \underline{108} \phantom{00} \\  24 \overline{)108} 4 \\  \underline{96} \phantom{00} \\  12 \overline{)24} 2 \\  \underline{24} \\  \hline  \text{Ans. } 6  \end{array}  $	<p>Therefore 12 is the greatest common measure of 108 and 132.</p> $  \begin{array}{r}  12 \overline{)78} 6 \\  \underline{72} \phantom{00} \\  6 \overline{)12} 2 \\  \underline{12} \\  \hline  \text{Ans. } 6  \end{array}  $
--	--

And the greatest common measure of all the numbers is 6.

## PROBLEM II.

To reduce a fraction to its lowest terms.

Find the greatest common measure by the last Problem, and divide both terms of the fractions by it, and the quotient will be the terms of the fraction required.

## EXAMPLE.

*Reduce  $\frac{72}{12}$  to its lowest terms.*

$  \begin{array}{r}  12 \overline{)72} 6 \\  \underline{72} \\  6 \overline{)12} 2 \\  \underline{12} \\  \hline  \text{Ans. } 6  \end{array}  $	<p>then <math>\frac{72}{12} = \frac{6}{2}</math> the ans.</p>
--	---

*Another method.*

Divide the terms of the given fraction by any digit above 1, that will divide them without a remainder, and these quotients again in the same way; and proceed in this manner as often as you can, and the last quotient will be the terms of the fraction required.

EX-

## EXAMPLE.

*Reduce  $2\frac{11}{12}$  to its lowest terms.*

$$\begin{array}{ccc} 3) & 4) & 9) \\ \frac{21}{12} & = \frac{11}{108} & = \frac{11}{12} = \frac{1}{1} \text{ Answer.} \end{array}$$

## PROBLEM III.

To reduce a whole number to the form of a fraction, which shall have a given denominator.

Multiply the whole number by the given denominator, and under that product place the said denominator.

## EXAMPLE.

*Reduce 7 to a fraction whose denominator shall be 3.*

$7 \times 3 = 21$ , hence  $\frac{21}{3}$  is the fraction required.

## PROBLEM IV.

To reduce a fraction of different denominations to equivalent ones, having a common denominator.

Multiply each numerator into all the denominators but its own, for a numerator to the required fraction, and all the denominators continually, for a common denominator.

## EXAMPLE.

Reduce  $\frac{1}{2}$ ,  $\frac{2}{3}$ , and  $\frac{1}{4}$ , to a common denominator.

$$1 \times 3 \times 4 = 12, \text{ numerator for } \frac{1}{2}.$$

$$2 \times 2 \times 4 = 16, \text{ ditto for } \frac{2}{3}.$$

$$3 \times 3 \times 2 = 18, \text{ ditto for } \frac{1}{4}.$$

---


$$2 \times 3 \times 4 = 24 \text{ common denominator.}$$


---

Therefore the fractions  $\frac{12}{24}$ ,  $\frac{16}{24}$ , and  $\frac{18}{24}$ , are the fractions required.

*Another method.*

I. Divide by any number that will divide two or more of the given denominators without a remainder, and set the quotients together, with the undivided numbers in a line below them; divide the second line as before, and proceed in this manner, till there is no two numbers that can be divided; then multiply all the divisors and quotients together for the common denominator.

II. Divide the common denominator by the denominator of each fraction; then multiply the quotient by each numerator, and the products will be the numerators of the fractions required; by this method you will have the least common denominator.

## EXAMPLE.

Reduce  $\frac{1}{2}$ ,  $\frac{2}{3}$ , and  $\frac{1}{4}$ , to fractions, having the least common denominator.

$$\begin{array}{r} 2) 2 \quad 3 \quad 4 \\ \hline 1 \quad 3 \quad 2 \end{array}$$

$$2 \times 1 \times 3 \times 2 = 12. \text{ The least common denominator.}$$

Then

Then $2 \overline{)12}$ $\underline{6}$ $\times 1$ $\underline{6 \text{ Num.}}$	$3 \overline{)12}$ $\underline{4}$ $\times 2$ $\underline{8 \text{ Num.}}$	$4 \overline{)12}$ $\underline{3}$ $\times 3$ $\underline{9 \text{ Num.}}$
--	---	---

Therefore the fractions are  $\frac{6}{12}$ ,  $\frac{8}{12}$ , and  $\frac{9}{12}$ .

### PROBLEM V.

To reduce a compound fraction to a single one.

Multiply all the numerators together, for the required numerator, and all the denominators together for the required denominator, and the fraction so found will be the answer.

#### EXAMPLE.

*Reduce  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{1}{4}$  to a single fraction.*

$$\text{Answer } \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}.$$

### PROBLEM VI.

To find the value of a vulgar fraction in known parts of a whole number,

I. Multiply the numerator by as many as will make one of the next inferior denomination, and divide the product by the denominator of the fraction.

II. If there is any remainder, multiply it by the next inferior denomination, and divide by the same denominator as before, and proceed in this manner with each, till you come to the lowest denomination, and the several quotients put together will show the known parts of integer, or whole numbers.

## EXAMPLE.

*What is the value of  $\frac{3}{16}$  of a pound sterling?*

$$\begin{array}{r}
 3 \\
 20 \\
 \hline
 13)60(4 \text{ Shillings.} \\
 52 \\
 \hline
 8 \\
 12 \\
 \hline
 13)96(7 \text{ Pence.} \\
 91 \\
 \hline
 5 \\
 4 \\
 \hline
 13)20(1\frac{7}{13} \\
 13 \\
 \hline
 7 \\
 \hline
 \end{array}$$

Answer 4s. 7 $\frac{1}{4}$ d. and  $\frac{7}{13}$  parts of a farthing.

*Note.* If the fraction is an improper one, divide the numerator by the denominator, and the quotient will be the highest denomination; and proceed with the remainder as before.

## PROBLEM VII.

To reduce a fraction of one denomination to the fraction of another.

I. If the fraction is to be reduced from a less denomination to a greater, multiply the denominator by all the denominations,

minations, from the given one to that which is sought; then place the numerator over the product, and it will be the fraction required.

II. But if from a greater to a less, multiply the numerator by all the denominations, as before, and place the given denominator under their product, and you will have the fraction required.

## EXAMPLE I.

*What part of a hundred weight is  $\frac{3}{4}$  of a pound?*

$$\text{Answer } \frac{3}{5 \times 28 \times 4} = \frac{3}{560} \text{ of an hundred weight.}$$

## EXAMPLE II.

*What part of a pound weight is  $\frac{3}{5}$  of an hundred weight?*

$$\text{Answer } \frac{3 \times 4 \times 28}{560} = \frac{336}{560} = \frac{3}{5} \text{ of a pound.}$$

## EXAMPLE III.

*What part of a penny is  $\frac{3}{4}$  of a pound sterling?*

$$\text{Answer } \frac{3 \times 20 \times 12}{5} = \frac{720}{5} \text{ of a penny.}$$

To add fractions together.

I. Reduce compound fractions to single ones, and mixed numbers to improper fractions; then reduce each fraction to the same denomination, and a common denominator.

II. Add all their numerators together, under the sum write the common denominator, and the fraction so represented will be their sum.

EX-

## EXAMPLE I.

*What is the sum of  $\frac{1}{2}$  and  $\frac{1}{3}$ ?*

$\frac{1}{2}$  and  $\frac{1}{3}$  reduced to a common denominator, are  $\frac{2}{3}$  and  $\frac{1}{3}$ ; then  $\frac{2}{3} + \frac{1}{3} = \frac{3}{3}$  their sum.

## EXAMPLE II.

*What is the sum of  $\frac{2}{3}$  of a pound, and  $\frac{2}{3}$  of a shilling, in parts of a penny?*

$\frac{2}{3}$  of a pound reduced to parts of a penny, is  $\frac{2 \times 20 \times 12}{3} = 160$ , and  $\frac{2}{3}$  of a shilling reduced to parts of a penny, is  $\frac{3 \times 12}{5} = 36$ , then  $160 + 36 = 196$  the answer in parts of a penny.

## EXAMPLE III.

*What is the sum of  $4\frac{2}{3}$  of a foot, and  $\frac{5}{8}$  of an inch, in parts of a foot?*

$4\frac{2}{3}$  reduced to an improper fraction, is  $\frac{14}{3}$  of a foot, and  $\frac{5}{8}$  of an inch, reduced to parts of a foot, is  $\frac{5}{8 \times 12} = \frac{5}{96}$ , and  $\frac{14}{3}$  reduced to a common denominator, is  $\frac{448}{96}$ , and  $\frac{5}{96}$ ; then  $\frac{448}{96} + \frac{5}{96} = \frac{453}{96}$  parts of a foot the answer.

## PROBLEM VIII.

To subtract one vulgar fraction from another.

Prepare the fractions as in addition, then subtract the numerators; write the common denominator under the difference, and you will have the difference required.

EX-

EXAMPLE I.

*What is the difference between  $\frac{2}{3}$  and  $\frac{1}{5}$ ?*

$\frac{2}{3}$  and  $\frac{1}{5}$  reduced, are  $\frac{4}{15}$  and  $\frac{3}{15}$ ; then  $\frac{4}{15} - \frac{3}{15} = \frac{1}{15}$ , the sum.

EXAMPLE II.

*What is the difference between  $\frac{2}{3}$  of a foot, and  $\frac{1}{5}$  of an inch, in parts of a foot?*

$\frac{1}{5}$  of an inch reduced to parts of a foot, is  $\frac{3}{5 \times 12} = \frac{3}{60}$   
then  $\frac{2}{3}$  and  $\frac{1}{5}$ , reduced to a common denominator, is  $\frac{40}{60}$   
and  $\frac{3}{60}$ ; therefore  $\frac{40}{60} - \frac{3}{60} = \frac{37}{60}$  parts of a foot is the difference.



PROBLEM X.

To multiply fractions or mixed numbers together.

I. Reduce mixed numbers to fractions, and each fraction to the same denomination.

II. Multiply their numerators together for the numerator of the fraction required; and their denominators together, for the required denominator; and the fraction so found will be the answer.

EXAMPLE I.

*What is the product of  $\frac{2}{3}$ , multiplied by  $\frac{1}{4}$ ?*

Answer  $\frac{2}{3} \times \frac{1}{4} = \frac{1 \times 2}{3 \times 4} = \frac{1}{6}$ .

EX-

## EXAMPLE II.

*What is the product of  $\frac{1}{5}$  of an inch, multiplied by  $\frac{1}{12}$  of a foot, in parts of a foot?*

$\frac{1}{5}$  of an inch reduced to parts of a foot, is  $\frac{3}{5 \times 12} = \frac{3}{60}$

then  $\frac{3}{60} \times \frac{1}{12} = \frac{1 \times 1}{40} = \frac{1}{40}$  the answer in parts of a foot.

When fractions are connected together by the sign  $\times$ , in order to have the product in its lowest terms, find a number that will divide one or more terms out of the numerators, and as many terms out of the denominators; then scratch out the divided terms, and write their quotients instead of them: proceed in the same manner with the quotients, and all the other undivided terms, till there is no terms that can be divided; then proceed as before, and you will have the answer in its lowest terms.

## EXAMPLE III.

*Multiply  $\frac{1}{3}$ ,  $\frac{5}{9}$ , and  $\frac{2}{10}$  continually together.*

$$\frac{1}{3} \times \frac{5}{9} \times \frac{2}{10} = \frac{1 \times 1 \times 1}{3 \times 1 \times 1} = \frac{1}{3}.$$

$\begin{array}{ccc} 1 & 1 & 1 \\ \cancel{3} & \cancel{9} & \cancel{10} \\ & 1 & 2 \\ & & 1 \end{array}$

## EXPLANATION.

It is easy to observe, that 9 in the denominator of  $\frac{5}{9}$ , will also measure 9 in the numerator of  $\frac{2}{10}$ ; therefore scratch out the terms 9, and 9, and put down their quotients 1 and 1: instead of them again, 5, the numerator of  $\frac{5}{9}$ , will measure 10, the denominator of  $\frac{2}{10}$ ; therefore scratch out 5, and 10, and

and put their quotients 1, and 2, instead of them, then the 2 now found, will also measure 2 in the numerator of  $\frac{2}{3}$ ; therefore scratch out 2, and 2, then there is no two terms that can be divided; and the remaining terms being multiplied together, the product  $\frac{1}{3}$  is in its lowest terms. This method will be frequently made use of in the following examples, where the fractions can be divided in that manner, as it is by far the readiest method.

## PROBLEM XI.

To divide one fraction by another.

Invert the divisor, and proceed as in multiplication.

### EXAMPLE I.

*Divide  $\frac{2}{3}$  by  $\frac{1}{2}$ .*

Answer,  $\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$ .

### EXAMPLE II.

*Divide  $\frac{2}{3}$  of a pound by  $\frac{2}{3}$  of a shilling, the quotient to be in parts of a pound.*

$\frac{2}{3}$  of a shilling reduced to parts of a pound, is  $\frac{8}{9 \times 20}$   
 $= \frac{2}{45} = \frac{2}{45}$  parts of a pound; then  $\frac{2}{3} \times \frac{45}{2} = \frac{1 \times 3}{1} = 3$ , the answer.

### EXAMPLE III.

*Divide  $\frac{2}{3}$  of a pound by  $\frac{2}{3}$  of a shilling.*

$\frac{2}{3}$  of a shilling reduced to parts of a pound, is  $\frac{2}{3 \times 20}$   
 $= \frac{2}{60} = \frac{1}{30}$ ; then  $\frac{2}{3} \times \frac{30}{1} = \frac{2 \times 10}{1} = 20$  parts of a pound the answer.

## EXAMPLE IV.

*Divide  $\frac{2}{3}$  of a foot by  $\frac{2}{7}$  of an inch.*

$\frac{2}{7}$  of an inch, reduced to parts of a foot, is  $\frac{5}{7 \times 12}$   
 $= \frac{5}{84}$ : then  $\frac{2}{3} \times \frac{84}{5} = \frac{56}{5}$  the answer.

## PROBLEM XII.

To resolve a question in simple proportion by vulgar fractions.

State the question, as directed before, and if any term be a whole number, reduce it to the form of a fraction, by writing one under it for a denominator; then invert the first term of the proportion, and multiply the three terms continually together, and the product will be the answer, which may be reduced to known parts, if necessary.

*Note.* If the first and third terms are not of the same denomination, they must be reduced to the same.

## EXAMPLE I.

*If  $\frac{1}{4}$  of a foot of wood cost 1s. 9d. what will  $5\frac{2}{3}$  of a foot cost?*

s. d.  
 1 9

12

—  
 21 reduced to pence.

—  
 Then  $\frac{1}{4} :: \frac{21}{1} :: 5\frac{2}{3} ::$

And  $\frac{4}{3} \times \frac{7}{1} \times \frac{17}{3} = \frac{476}{3} = 158\frac{2}{3} = £ 0 13 2\frac{1}{2}$ , and  $\frac{2}{3}$  of a farthing the answer.

EX-

EXAMPLE II.

*If a man be  $2\frac{1}{2}$  days in digging any quantity of earth, of 3 degrees hardness, what time will he be in digging the same quantity of the foundation for a building of 7 degrees hardness?*

Here it is evident, that the hardness will be as the time, when the quantity dug is the same; for if double the hardness, it will take double the time, and treble the hardness, treble the time, &c.

Then  $\frac{1}{3} : 2\frac{1}{2} :: \frac{7}{7} :$  and  $\frac{1}{3} \times \frac{5}{2} \times \frac{7}{7} = \frac{35}{6} = 5\frac{5}{6}$  days the ans.

EXAMPLE III.

*If a labourer can dig  $5\frac{2}{3}$  solid yards, of 3 degrees hardness, in a certain time, how many solid yards of earth will he dig of a ditch, that is 5 degrees hardness, in the same time?*

Here it is evident, that the quantity dug, will be inversely as the degrees of hardness, when the time is the same; for in equal terms, if double the hardness, only half the quantity will be dug; and treble the hardness,  $\frac{1}{3}$  will be dug, &c.

Then  $\frac{1}{3} : 5\frac{2}{3} :: \frac{1}{5} :$  and  $\frac{3}{1} \times \frac{17}{3} \times \frac{1}{5} = \frac{17}{5} = 3\frac{2}{5}$  yards the answer.

In this Example, I invert the terms of hardness; that is, instead of 3 or  $\frac{2}{3}$  degrees of hardness, I write  $\frac{1}{3}$ ; and instead of  $\frac{2}{3}$ , I write  $\frac{1}{5}$ . As these terms are not direct, the same is to be understood in any of the following Examples, where any inverse terms occur.

## PROBLEM XIII.

To resolve a question in compound proportion,  
by vulgar fractions.

State the question, as directed before ; if any term is a whole number, reduce it to the form of a fraction, as in simple proportion; then invert all the terms of your divisors, and multiply them and all the other terms continually together, and the product will be the answer.

*Note.* That all similar terms must be reduced to the same denomination.

## EXAMPLE I.

*It was observed, that 9 bricklayers, in  $14\frac{2}{7}$  days, built a wall to the height of  $4\frac{1}{2}$  feet, how many bricklayers must be employed to complete the remainder, which is 7 feet, in  $6\frac{2}{3}$  days?*

causes    eff.    causes    eff.

$$\frac{9}{1} \times 14\frac{2}{7} \div 4\frac{1}{2} :: x \times 6\frac{2}{3} \div 7.$$

$$\frac{9}{1} \times \frac{100}{7} \times \frac{2}{9} \times \frac{3}{7} \times \frac{7}{1} = 30 \text{ bricklayers the answer.}$$

## EXAMPLE II.

*If 12 inches long, and 12 inches broad, will make a square foot, how much in length, that is  $9\frac{1}{4}$  wide, will make a square foot?*

causes    eff.    causes    eff.

$$\frac{12}{1} \times \frac{12}{1} \div \frac{1}{1} :: 9\frac{1}{4} \times x \div \frac{1}{1}.$$

$$\frac{12}{1} \times \frac{12}{1} \times \frac{1}{1} \times \frac{4}{29} \times \frac{1}{1} = \frac{12^2}{13} = 14\frac{10}{13} \text{ inches the answer.}$$

EX-

EXAMPLE III.

If 12 inches long, and 12 inches broad, and 12 inches deep, make a solid foot, how many solid feet will  $32\frac{1}{2}$  long,  $16\frac{1}{2}$  inches broad, and  $10\frac{1}{4}$  deep make?

$$\begin{array}{ccccccc} & \text{causes} & & \text{eff.} & & \text{causes} & & \text{eff.} \\ & \overbrace{\frac{1}{12} \times \frac{1}{12} \times \frac{1}{12}} & & \div \frac{1}{1} & & \overbrace{32\frac{1}{2} \times 16\frac{1}{2} \times 10\frac{1}{4}} & & \div x. \\ \frac{1}{12} \times \frac{1}{12} \times \frac{1}{12} \times \frac{1}{1} \times \frac{6\frac{1}{2}}{2} \times \frac{1}{3} \times \frac{1}{4} & = & \frac{1 \cdot 3 \cdot 6 \cdot 2 \cdot 5 \cdot 5}{2 \cdot 4 \cdot 1 \cdot 4 \cdot 7 \cdot 2} & = & 3\frac{1 \cdot 2 \cdot 5 \cdot 3}{4 \cdot 1 \cdot 4 \cdot 7 \cdot 2} & \text{solid} \\ & & & & & & & \text{feet the answer.} \end{array}$$

EXAMPLE IV.

If 248 men, in  $5\frac{1}{2}$  days, of 11 hours each, can dig a trench, 7 degrees hardness,  $232\frac{1}{2}$  yards long,  $3\frac{2}{3}$  wide, and  $2\frac{1}{2}$  deep, how many days, of 9 hours long, will 24 men be in digging a trench of 4 degrees hardness,  $337\frac{1}{2}$  yards long,  $5\frac{1}{2}$  wide, and  $3\frac{1}{2}$  deep?

In this question, the quantity dug will be as the number of men, days, and hours, and inversely as the degrees of hardness.

$$\begin{array}{ccccccc} & \text{causes} & & \text{effect} & & \text{causes} & & \text{effect} \\ & \overbrace{\frac{248}{1} \cdot 5\frac{1}{2} \cdot \frac{11}{1} \cdot \frac{1}{7}} & & \div \overbrace{232\frac{1}{2} \cdot 3\frac{2}{3} \cdot 2\frac{1}{2}} & & \overbrace{\frac{24}{1} \cdot 9 \cdot \frac{4}{1} \cdot \frac{1}{7}} & & \div \overbrace{337\frac{1}{2} \cdot 5\frac{1}{2} \cdot 3\frac{1}{2}} \\ & & & & & & & \\ & \frac{248}{1} \cdot \frac{11}{1} \cdot \frac{11}{1} \cdot \frac{1}{7} \times \frac{1}{408} \cdot \frac{3}{11} \cdot \frac{3}{7} \times \frac{1}{24} \cdot \frac{1}{9} \cdot \frac{4}{1} \times \frac{678}{1} \cdot \frac{28}{8} \cdot \frac{1}{2} & & & & & & \\ & & & \frac{11 \cdot 3 \cdot 3}{3 \cdot 8} & & & & \frac{678 \cdot 28}{8} \end{array}$$

Then  $11 \times 3 \times 4 = 132$  the answer.

*Note.* That the dots placed between the terms in this Example, signifies the same as  $\times$ .

## DECIMAL FRACTIONS.

### DEFINITION.

**D**ECIMAL Fractions are such as have an unit, with any number of ciphers joined to it for their denominator; thus,  $\frac{3}{10}$ ,  $\frac{2}{100}$ , and  $\frac{35}{1000}$ , are decimal fractions: here the integer is always supposed to be divided into 10, 100, 1000, &c. equal parts; or this, which amounts to the same thing, it is divided into 10 equal parts, and each of these again into 10, &c.

### NOTATION.

As whole numbers increase from unity in a tenfold proportion to the left hand, or this, which is the same thing, they decrease from the left hand to unity in a tenfold proportion; so decimals also decrease in a tenfold proportion from unity towards the left hand; therefore, in order to distinguish decimals from whole numbers, a point is always prefixed to the decimals, which also shows the first place of decimals from unity, which observe in the following table:

Whole

Whole numbers.								Decimals.				
3	4	5	6	4	9	3	4	7	8	3	4	5
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
Tens of millions								Tenth parts				
Millions								Hundredth parts				
Hundreds of thousands								Thousandth parts				
Tens of thousands								Tens of thousandth parts				
Thousands								Hundredths of thousandth parts				
Hundreds												
Tens												
Units												

## PROBLEM I.

To add decimals or mixed numbers together.

Set down each number in its respective place, under those figures of the same name with itself, and add them together, as in common addition.

Ex. I.	Ex. II.
738549	734·965
·003872	·00854
·498547	49·372
·035024	530·4265
<hr/>	<hr/>
Sum 1·275992	Sum 1314·77204
<hr/>	<hr/>

## PROBLEM

## PROBLEM II.

To subtract one decimal from another, or mixed numbers.

Place the numbers as in addition, and perform the operation, as in subtraction of integers.

Ex. I.

From	7854
Take	49357
<hr/>	
Rem.	29183
<hr/>	

Ex. II.

From	4873465
Take	0538
<hr/>	
Rem.	4868085
<hr/>	

## PROBLEM III.

To multiply decimals, or whole numbers and decimals.

I. Proceed as in whole numbers, then cut off as many places from the right hand of the product, as there are places in the multiplier and multiplicand taken together; then the number of places cut off, will show the decimal places of the product.

II. If the number of figures in the product, be less than the places in the multiplier and multiplicand taken together, you must add as many ciphers to the left hand of the product, to make them out.

EX-

Ex. I.

Multiply .9087  
By .852

18174  
45435  
72696

Product .7742124

Ex. II.

Multiply .25483  
By .246

152898  
101932  
50966

Product .06268818

Ex. III.

Multiply .003479  
By 5.081

3479  
27832  
17395

Product .017676799

Ex. IV.

Multiply 231.7  
By 2.016

13902  
2317  
4634

Product 467.1072

## How to contract the work in large decimals.

I. Write the units place of the multiplier under that place of decimals in the multiplicand, whose place you would reserve in the product: write the other figures in the multiplier in a contrary order.

II. Begin with the first figure of the multiplier towards the left hand, and multiply it by the next figure towards the right of it in the multiplicand, if any; and if this product is 5, or above 5, in the number of tens in the product, you must carry 1 more than there is tens in the last product to the next product: then set down the overplus above the tens for the first figure, and carry the tens to the next place,

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and

and proceed through the line, as in common multiplication.

III. Proceed with every other line in the same manner, but observe to place the first figure of every line directly under the first place of the last line, and the product so found will be the answer.

### EXAMPLE I.

*What is the product of .9087, multiplied by .852, and to retain four places of decimals?*

contracted	common way
.9087	.9087
258.0 multiplier inverted	.852
<hr/>	<hr/>
7270=908×8+6	18 174
454=90×5+4	454 35
18=9×2+0	7269 6
<hr/>	<hr/>
Product .7742	.7742 124
<hr/>	<hr/>

### EXPLANATION.

I. The multiplier being placed as directed, begin with the first figure of the multiplier, viz. 8, then 7 is the next figure in the multiplicand on the right hand of it; then I multiply 8 by 7, which is 56, and 6 being more than 5 above 5 tens, therefore I carry 6: again,  $8 \times 8 = 64$ , and 6 carried makes 70; I set down nothing for the first figure, and carry 7 to the next, and proceed through the line, and find its product to be 7270.

II. I begin with the next figure 5 in the same manner, and multiply it by 8, which is the next figure in the multi-

multiplicand on the right of it, and the product is 40; but as 40 is not 5 above 4 tens, I only carry 4, and say, 5 times 0 is 0, and 4 carried is 4, which is the first figure of the second, then proceed, as in the first line, with each product, and the sum, viz., 7742, is the product true to the last figure.

### EXAMPLE II.

*What is the product of 2.38645, multiplied by 8.2175, so as to have two places of decimals in the product?*

	or this
2.38645	8.2175
5712.8	683.2
<hr/>	<hr/>
1909	1643
48	246
2	66
1	5
<hr/>	<hr/>
Answer 19.60	Answer 19.60
<hr/>	<hr/>

It is obvious from this Example, that it would be of no use to write more than one figure in the multiplier on the left of the multiplicand, as those figures towards the left hand in the multiplier, are made no use of.

### PROBLEM IV.

To divide a decimal, or mixed number, by a decimal, or mixed number.

I. Divide, as in whole numbers, and cut off as many places from the right hand of the quotient for decimals, as the decimal places in the dividend exceed those of the divisor.

II. But if the places of the quotient are not so many as the excess of the decimal places of the dividend above those of the divisor, you must supply that defect by prefixing ciphers on the left hand of it.

III. And if there is a remainder, annex ciphers to it, and by this means you may carry the quotient as far as is requisite.

Ex. I.

*Divide 43.95 by 5*  

$$\begin{array}{r} 5 \overline{)43.95} \\ \hline \end{array}$$

Answer  $\underline{\underline{8.79}}$

Ex. II.

*Divide 4.9536 by 6*  

$$\begin{array}{r} 6 \overline{)4.9536} \\ \hline \end{array}$$

Answer  $\underline{\underline{8.256}}$

Ex. III.

*Divide 53.87 by 3.5*  

$$\begin{array}{r} 3 \times 7 = 21 \\ 5 \overline{)53.870} \\ \hline \end{array}$$

$7 \overline{)10774000}$

$15.39142 +$   
 Ans.

Ex. IV.

*Divide .3862409 by 21*  

$$\begin{array}{r} 3 \times 7 = 21 \\ 3 \overline{).3862409000} \\ \hline \end{array}$$

$7 \overline{)1287469666 +}$

$.0183924239 +$   
 Ans.

Ex. V.

*Divide 4.368 by .0078*  

$$\begin{array}{r} .0078 \overline{)4.368(56} \\ \hline \end{array}$$

390

468

468

Ex. VI.

*Divide .04575 by 27*  

$$\begin{array}{r} 27 \overline{).04575(00169(3} \\ \hline \end{array}$$

27

187

162

253

243

100

81

19 &c.

To

### To contract division into large decimals.

I. Write down the divisor and quotient, also find the first product, and subtract it as usual: then the first figure of the quotient must always possess the same place in the quotient as that figure in the dividend under which units place in the product stands; that is, if units place in the product stand under integers in the dividend, you will have as many places of integers in the quotient, as there are places from the units place in the dividend to that place above which units place of the product stands.

II. If the units place of the product stand under decimals in the dividend, your first quotient figure will be the same place of decimals in the quotient, as the units place in the product stands from the first place of decimals in the dividend; consequently, you must prefix ciphers to make it out to that place.

III. Cut off as many places from the left of your divisor as will be equal to the number of places that you intend your quotient figures to consist of, for a divisor; then cut off as many places from the right of the remainder, as the remaining figures cut off or thrown out of the divisor; then that part of the remainder, to the left hand, you must esteem a dividend.

IV. In multiplying by your next quotient figure, you must omit one place in your divisor to the right hand, but observe to carry what would arise by multiplying the quotient figure, by the figure omitted in your divisor, as in contracted multiplication of decimals.

V. Pro-

V. Proceed in this manner through the whole, esteeming every succeeding remainder a dividend, and for every remainder omit a figure to the left hand of the divisor, but add the carriage that would arise from the figure omitted, to the first figure of each product, as before, and you will have your desire.

## EXAMPLE I.

*Divide 36.96630854321 by 4.3685, and retain three places of decimals in the quotient.*

$$\begin{array}{r}
 4.3685 \overline{)36.96630855321(8.462} \\
 \underline{34.948} 0 \\
 2018 \overline{)3} \\
 \underline{1747} = 436 \times 4 + 3 \\
 271 \\
 \underline{262} = 43 \times 6 + 4 \\
 9 \\
 \underline{9} = 4 \times 2 + 1 \\
 0
 \end{array}$$

Proof  $\overline{36966}$  by addition.

In this Example, the units place of the first product falls under the first place of integers in the dividend; consequently, there is only one place of integers in the quotient; then, in order to have three places of decimals in your quotient, you must take four places from the left of your divisor, and there is one place remains: also cut off or throw away one place of the remainder, and 2018 will be esteemed a dividend; then proceed as directed.

EX.

EXAMPLE II.

*Divide 574·34782 by 6134775, and retain two places of decimals in the quotient.*

$$\begin{array}{r}
 \cdot 61347 \overline{) 574 \cdot 347 936 (936 \cdot 21} \\
 \cdot \cdot \cdot \cdot \quad 5 \cdot 52129 \overline{) 75} \\
 \hline
 \quad \quad \quad 22217 \overline{) 61} \\
 \quad \quad \quad 18404 \\
 \hline
 \quad \quad \quad \quad 3813 \\
 \quad \quad \quad \quad 3680 \\
 \hline
 \quad \quad \quad \quad \quad 133 \\
 \quad \quad \quad \quad \quad 123 \\
 \hline
 \quad \quad \quad \quad \quad \quad 10 \\
 \quad \quad \quad \quad \quad \quad \quad 6 \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad \quad 4
 \end{array}$$

In this Example, the units place in the product falls under the third place of integers in the dividend, therefore you will have three places of integers in the quotient; consequently you must cut off five places from the divisor, in order to have two places of decimals.

EXAMPLE III.

*Divide 75·434748 by 61377·5, and preserve four places in the quotient, besides the prefixed ciphers.*

$$\begin{array}{r}
 6137 \overline{) 75 \cdot 43 4748 (001229} \\
 \cdot \cdot \cdot \quad 61 \ 37 \overline{) 7 \cdot 5} \\
 \hline
 \quad \quad \quad 14 \ 05 \overline{) 7 \ 2} \\
 \quad \quad \quad 12 \ 27 \\
 \hline
 \quad \quad \quad \quad 178 \\
 \quad \quad \quad \quad 123 \\
 \hline
 \quad \quad \quad \quad \quad 55 \\
 \quad \quad \quad \quad \quad 55 \\
 \hline
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

In

In the last Example, because the units place in the product stands under the third place of decimals, therefore I prefix two ciphers, which makes the first quotient figure out to three places of decimals, and proceed as in the former Example.

## PROBLEM V.

To reduce a vulgar fraction to a decimal fraction.

Add ciphers at pleasure to the numerator of decimal places, and divide by the denominator as far as it is necessary, as in division of decimals.

Ex. I.

*What is the decimal of  $\frac{1}{2}$ ?*

$$\begin{array}{r} 2 \overline{)1\cdot0} \\ \text{Ans. } \cdot 5 \end{array}$$

Ex. II.

*What is the decimal of  $\frac{1}{4}$ ?*

$$\begin{array}{r} 4 \overline{)1\cdot00} \\ \text{Ans. } \cdot 25 \end{array}$$

Ex. III.

*What is the decimal of  $\frac{1}{4}$ ?*

$$\begin{array}{r} 4 \overline{)3\cdot00} \\ \text{Ans. } \cdot 75 \end{array}$$

Ex. IV.

*What is the decimal of  $\frac{5}{8}$ ?*

$$\begin{array}{r} 8 \overline{)5\cdot000} \\ \text{Ans. } \cdot 625 \end{array}$$

Ex-

Ex. V.

*What is the decimal of  $\frac{1}{3}$ ?*

$$\begin{array}{r} 3 \overline{)2.0000} \\ \text{Ans. } .6666 \text{ \&c.} \end{array}$$

Ex. VI.

*What is the decimal of  $\frac{1}{4}$ ?*

$$\begin{array}{r} 54 = 6 \times 9 \\ 6 \overline{)3.00} \\ 9 \overline{)500} \\ \text{Ans. } .055 \text{ \&c.} \end{array}$$

Ex. VII.

*What is the decimal of  $\frac{2}{3}$ ?*

$$\begin{array}{r} 3 \overline{)22.000} \\ \text{Ans. } 7.333 \text{ \&c.} \end{array}$$

Ex. VIII.

*What is the decimal of  $\frac{7}{11}$ ?*

$$\begin{array}{r} 31 = 3 \times 7 \\ 3 \overline{)9.00000} \\ 7 \overline{)26.33333 \text{ \&c.}} \\ \text{Ans. } 3.76190+ \end{array}$$

## PROBLEM VI.

To reduce the known parts of any integer to a decimal fraction.

I. Begin at the lowest denomination, and divide it by as many as will make one of the next superior denomination, and add to the quotient the next superior denomination, if any, of the given integer.

II. Proceed in this manner to the highest place, observing to point out the decimal parts at each operation, and you will have the answer.

## EXAMPLE I.

*Reduce 15s. 9d. to the decimal of a shilling.*

$$\begin{array}{r} 4 \overline{) 3 \cdot 00} \\ 12 \overline{) 9 \cdot 75} \\ \hline \text{Ans. } 15 \cdot 8125 \end{array}$$

## EXAMPLE II.

*Reduce 14s. 6d. to the decimal of a pound.*

$$\begin{array}{r} 12 \overline{) 6 \cdot 0} \\ 2,0 \overline{) 14 \cdot 5,0} \\ \hline \text{Ans. } \cdot 725 \end{array}$$

## EXAMPLE III.

*Reduce 2ft. 3in. to the decimal of a foot.*

$$\begin{array}{r} 12 \overline{) 3 \cdot 00} \\ \hline \text{Ans. } 2 \cdot 25 \end{array}$$

## PROBLEM VII.

To reduce a decimal to the known parts of an integer.

I. Multiply the decimal by the next inferior denomination, and cut off the decimal parts.

II. Multiply the decimal parts by the next inferior denomination, cut off the decimals as before, proceed in this manner to the lowest denomination, and the parts on the left hand of the decimals will be the answer.

EX-

## EXAMPLE I.

*What is the known parts of  $\cdot 725$  of a pound sterling?*

$$\begin{array}{r} 7 \cdot 25 \\ 20 \\ \hline 14 \cdot 500 \\ 12 \\ \hline 6 \cdot 0 \end{array}$$

Ans. 14s. 6d.

## EXAMPLE II.

*What is the value of  $2 \cdot 25$  of a foot?*

$$\begin{array}{r} 2 \cdot 25 \\ 12 \\ \hline 3 \cdot 00 \end{array}$$

Ans. 2ft. 3in.

I shall here end decimal arithmetic, and show its application more particularly to mensuration; as my design in this book is not to write a book of arithmetic, but only to explain the most useful rules, that the artist may have every possible advantage in its application, as it may sometimes fall in his way.

## DUODECIMALS.

### DEFINITION.

**D**UODECIMALS are so called, because they decrease continually by twelve, from the place of feet towards the right hand.

### NOTATION.

Feet are marked thus, f.; inches, also called firsts, thus, i; seconds, thus, ii; thirds, thus, iii; fourths, thus, iv; and fifths, thus, v; &c.

Thus,  $\overset{\text{iii}}{4}$  signifies 4 thirds;  $\overset{\text{i}}{3}$  signifies 3 inches, or 3  
 $\overset{\text{ii}}{\text{firsts}}$ ;  $\overset{\text{f.}}{5}$  signifies 5 seconds; and 12 signifies 12 feet.

### PROBLEM VIII.

To multiply duodecimals together.

I. Write the multiplier under the multiplicand.

II. Multiply the first denomination in the multiplier on the right hand, by every denomination in the multiplicand, observing to carry one for every 12, out of each respective product, to the next superior place.

III. Put

III. Put the first place of each respective product directly under that place of the multiplier by which you multiply with, as in common multiplication, and each column being added together, will give the product.

IV. Add the characters denoting the two last places in the multiplier and multiplicand together, and that will be the character denoting the last place of the product from the place of feet.

EXAMPLE I.

*Multiply 6f. 9i by 3f. 6i.*

	f.	i	
	6	9	
	3	6	
	<hr/>		
	3	4	6
	20	3	
	<hr/>		
Ans.	23	7	6
	f.	i	ii

In this Example there is only one place of duodecimals in each factor, therefore there is two places of duodecimals in the product; that is, firsts or i added to firsts or i, gives ii or seconds for the first character towards the right hand of the product.

EXAMPLE II.

*Multiply 6f. 5i 4ii by 3f. 6i.*

	f.	i	ii	
	6	5	4	
		3	6i	
	<hr/>			
	3	2	8	0
	19	4	0	
	<hr/>			
Ans.	22	6	8	0
	f.	i	ii	iii

In

In the last Example, the two last places of the factors are marked thus, ii and i, which added together, is iii, and signifies there is three places of duodecimals in the product, from the first place of feet; therefore the first place on the right hand is thirds.

## EXAMPLE III.

Multiply 15f. 6i 3ii by 12f. 8i 9ii.

	f.	i	ii	
	15	6	3	
	12	8	9ii	
<hr/>				
	11	7	8	3
10	4	2	0	
186	3	0		
<hr/>				
Ans.	197	6	9	8 3
	f.	i	ii	iii iv

In this Example, the two last places of the factors are marked thus, ii and ii; these added together, will show that there is four places of duodecimals in the product, therefore the first place from the right hand is fourths.

## EXAMPLE IV.

Multiply 5f. 4i 3ii 5iii by 4f. 8i 3ii.

	f.	i	ii	iii	
	5	4	3	5	
		4	8	3ii	
<hr/>					
	1	4	0	10	3
3	6	10	3	4	
21	5	1	8		
<hr/>					
Ans.	25	1	4	0	2 3
	f.	i	ii	iii	iv v

EX-

### EXAMPLE V.

**What is the product of 4i 3ii 0iii 0iv 5v by 4ii 3iii 5iv?**

						i	ii	iii	iv	v	
						4	3	0	0	5	
								4	3	5iv	
						<hr/>					
						1	9	3	0	2	1
						1	0	9	0	1	3
						1	5	0	0	1	8
						<hr/>					
Ans.	0	0	1	6	2	6	4	9	5	1	
	f.	i	ii	iii	iv	v	vi	vii	viii	ix	

### EXAMPLE VI.

*What is the product of 4f. 3i 2ii 8iii 9iv 5v 3vi by 3i 0ii 5iii 0iv 0v 6vi 2vii?*

[illegible]

When there is a great number of feet in the multiplier, the best way is by the rule of practice, as follows:

**Mul-**

Multiply the feet together, as in common multiplication; and for the odd inches, &c. take the aliquot parts.

Thus, for 1i take  $\frac{1}{12}$  of a foot, or 1ii take  $\frac{1}{12}$  of 1i, &c.

for 2	—	$\frac{1}{6}$
for 3	—	$\frac{1}{4}$
for 4	—	$\frac{1}{3}$
for 5	—	$\frac{1}{4}$ and $\frac{1}{6}$
for 6	—	$\frac{1}{2}$
for 7	—	$\frac{1}{2}$ and $\frac{1}{12}$
for 8	—	$\frac{1}{2}$ and $\frac{1}{6}$
for 9	—	$\frac{1}{2}$ and $\frac{1}{4}$
for 10	—	$\frac{1}{2}$ and $\frac{1}{3}$
for 11	—	$\frac{2}{3}$ and $\frac{1}{6}$

### EXAMPLE I.

Multiply 240f. 10i 8ii by 9f. 4i 6ii.

	f.	i	ii	
	240	10	8	
	9	4	6	
	<hr/>			
	2168	0	0	
4 = $\frac{1}{2}$	-	-	80	3 6 8
6 = $\frac{1}{6}$	-	-	10	0 5 4
	<hr/>			
Ans.	2258	4	0 0	
	<hr/>			

If the feet in both the multiplicand and multiplier be large numbers,

Multiply the feet only into each other: then, for the inches and seconds in the multiplier, take parts of the multiplicand; and for the inches and seconds of the multiplicand, take aliquot parts of the feet only in the multiplier; and the sum of all will be the product.

EX.

EXAMPLE II.

Multiply 368f. 7i. 5ii by 137f. 8i 4ii.

	f.	i	ii
368	7	5	
137	8	4ii	
<hr/>			
	2576		
	1104.		
	368.		
6i = $\frac{1}{2}$	- -	184	3 8 6
2i = $\frac{1}{4}$	- -	61	5 2 10
4ii = $\frac{1}{8}$	- -	10	2 10 5 8
6i = $\frac{3}{4}$	- -	68	6 . . .
1i = $\frac{1}{2}$	- -	11	5 . . .
4ii = $\frac{1}{4}$	- -	3	9 8 . .
1ii = $\frac{1}{8}$	- -		11 5 . .
<hr/>			
Ans. 50756		7 10 9 8	

INVOLUTION,

OR,

RAISING OF POWERS.

DEFINITION.

**A** POWER is a number produced by multiplying any given number continually by itself a certain number of times.

Any number is called the first power of itself; if it be multiplied by itself, the product is called the second power,

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and sometimes the square; if this be multiplied by the first power again, the product is called the third power, and sometimes the cube; and if this be multiplied by the first power again, the product is called the fourth power, and so on; that is, the power is denominated from the number which exceeds the multiplications by 1.

Thus: 3 is the first power of 3.

$3 \times 3 = 9$  is the second power of 3.

$3 \times 3 \times 3 = 27$  is the third power of 3.

$3 \times 3 \times 3 \times 3 = 81$  is the fourth power of 3.

## A TABLE

OF THE

### FIRST FOUR POWERS OF NUMBERS.

1st power	1	2	3	4	5	6	7	8	9
2d power	1	4	9	16	25	36	49	64	81
3d power	1	8	27	64	125	216	343	512	729
4th power	1	16	81	256	625	1296	2401	4096	6561

### NOTATION.

The number which exceeds the multiplications by 1, is called the index, or exponent of the power; so the index of the first power is 1, that of the second power is 2, that of the third is 3, and so on.

Powers are commonly denoted by writing their indices above the first power: so the second power of three is denoted

noted thus,  $3^2$ ; the third power thus,  $3^3$ ; the fourth power thus,  $3^4$ ; and so on: also the sixth power of 503 thus,  $503^6$ .

Involution is the finding of powers; to do which, from their definition there evidently comes this

## PROBLEM IX.

To raise a given number to any given power required.

I. Multiply the given number, or first power, continually by itself, till the number of multiplications be 1 less than the index of the power to be found, and the last product will be the power required.

II. But because fractions are multiplied by taking the products of their numerators and of their denominators, they will be involved by raising each of their terms to the power required: and if a mixed number be proposed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

Ex. I.

Ex. II.

*What is the square of 45?*

*What is the square of .027?*

$$\begin{array}{r}
 45 \text{ 1st power.} \\
 45 \\
 \hline
 225 \\
 180 \\
 \hline
 2025 = 45^2
 \end{array}$$

$$\begin{array}{r}
 .027 \\
 .027 \\
 \hline
 189 \\
 54 \\
 \hline
 .000729 = .027^2
 \end{array}$$

## Ex. III.

*What is the cube of 3.5?*

$$\begin{array}{r}
 3.5 \\
 3.5 \\
 \hline
 175 \\
 105 \\
 \hline
 12.25 \\
 3.5 \\
 \hline
 6125 \\
 3675 \\
 \hline
 42.875 = 3.5^3
 \end{array}$$

## Ex. IV.

*What is the fourth power of 5.1?*

$$\begin{array}{r}
 5.1 \\
 .5.1 \\
 \hline
 51 \\
 255 \\
 \hline
 260.1 = 5.1^2 \\
 26.01 \text{ ditto.} \\
 \hline
 2601 \\
 15606 \\
 5202 \\
 \hline
 676.5201 = 5.1^4
 \end{array}$$

## EXAMPLE V.

The square of  $\frac{2}{3}$  is  $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ .

## EXAMPLE VI.

The cube of  $\frac{5}{9}$  is  $\frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} = \frac{125}{729}$ .

## EXAMPLE VII.

The square of  $3\frac{2}{3}$  or  $\frac{11}{3}$  is  $\frac{11}{3} \times \frac{11}{3} = \frac{121}{9} = 11\frac{14}{9} = 11.56$ .

## EVOLUTION,

OR

## EXTRACTION OF ROOTS.

*DEFINITION.*

**T**HE root of any given number, or power, is such a number, as being multiplied by itself a certain number of times, will produce the power; and it is denominated the first, second, third, fourth, &c. root respectively, as the number of multiplications made of it to produce the given power is 0, 1, 2, 3, &c. that is, the name of the root is taken from the number which exceeds the multiplications by 1, like the name of the power in involution.

*NOTATION.*

Roots are sometimes denoted by writing  $\sqrt{\phantom{x}}$  before the power, with the index of the root against it: so the third root of 50 is  $\sqrt[3]{50}$ , and the second root of it is  $\sqrt{50}$ , the index 2 being omitted; which index is always understood when a root is named or written without one. But if the power be expressed by several numbers with the sign  $+$  or  $-$ , &c. between them, then a line is drawn from the top of the sign of the root or radical sign, over all

all the parts of it; so the third root of  $47 - 15$  is  $\sqrt[3]{47-15}$ . And sometimes roots are designed like powers, with the reciprocal of the index of the root above the given number. So the root of 3 is  $3^{\frac{1}{3}}$ ; the root of 50 is  $50^{\frac{1}{3}}$ , and the third root of it is  $50^{\frac{1}{9}}$ ; also the third root of  $47-15$  is  $\sqrt[3]{47-15^{\frac{1}{3}}}$ . And this method of notation is justly preserved in the modern algebra; because such roots, being considered as fractional powers, need no other directions for any operations to be made with them, than those of integral powers.

A number is called a complete power of any kind, when its root of the same kind can be accurately extracted; but if not, the number is called an imperfect power, and its root a surd or irrational quantity. So 4 is a complete power of the second kind, its root being 2; but an imperfect power of the third kind, its third root being a surd quantity, which cannot be accurately extracted.

Evolution is the finding of the roots of numbers, either accurately, or in decimals to any proposed degree of accuracy.

The power is first to be prepared for extraction, or evolution, by dividing it, by means of points or commas, from the place of units to the left hand in integers, and to the right in decimal fractions, in periods, containing each as many places of figures as are denoted by the index of the root, if the power contain a complete number of such periods; that is, each period to have two figures for the square root, three for the cube root, four for the fourth root, and so on. And when the last period in decimals is not complete, ciphers are added to complete it.

*Note.* The root will contain just as many places of figures, as there are periods or points in the given power; and they will

will be integers, or decimals respectively, as the periods are so from which they are found, or to which they correspond; that is, there will be as many integers or decimal figures in the root, as there are periods of integers or decimals in the given number.

## PROBLEM X.

To extract the square root.

I. Having divided the given number into periods of two figures each, find, from the table of powers in page 138, or otherwise, a square number, either equal to, or the next less than the first period, which subtract from it, and place the root of the square on the right of the given number, after the manner of a quotient in division, for the first figure of the root required.

II. To the remainder annex the second period for a dividend; and on the left thereof write the double of the root already found, after the manner of a divisor.

III. Find how often the divisor is contained in the dividend, wanting its last figure on the right hand; place that number for the next figure in the quotient, and on the right of the divisor, as also below the same.

IV. Multiply the whole increased divisor by it, placing the product below the dividend, which subtract from it, and to the remainder bring down the next period, for a new dividend; to which, as before, find a divisor by doubling the figures already found in the root; and from these find  
the

the next figure of the root, as in the last article; and continue the operation still in the same manner till all the periods be used, or as far as you please.

*Note.* Instead of doubling the root, to find the new divisors, you may add the last divisor to the figure below it.

To prove the work, multiply the root by itself, and to the product add the remainder, and the sum will be the given number.

### EXAMPLE 1.

*What is the root of 17·30,56?*

$$\begin{array}{r}
 17\cdot30,56(4\cdot16 \\
 16 \\
 \hline
 \begin{array}{r|l}
 81 & 130 \\
 1 & 81 \\
 \hline
 \end{array} \\
 \hline
 \begin{array}{r|l}
 826 & 4956 \\
 6 & 4956 \\
 \hline
 \end{array}
 \end{array}$$

### EXPLANATION.

Having divided the given number into three periods, namely, 17, and 30, and 56, I find that 16 is the next square to 17, the first period, which set below, and subtracting, 1 remains, to which bring down 30, the next period, makes 130 for a dividend: then 4, the root of 16, is set on the right hand of the given number for the first figure of the root, and its double, or 8, on the left of the dividend for the first figure of the divisor; which being once contained in 13, the dividend wanting its last figure, gives 1 for the next figure of the root, which 1 is accordingly set in the root, making 4·1; and

and in the divisor making 81, as also below the same. These multiplied make also 81, set below the dividend, and subtracting, we have 49 remaining, to which the last period 56 being brought down, we have 4956 for the new dividend. Then, for a new divisor, either double the root 4·1, or else, which is easier, to the last divisor add the figure 1 standing below it, and either way gives 82 for the first part of the new divisor. This 82 is 6 times contained in 495, and therefore 6 is the next figure to set in the root and in the divisor, as also below the same; which being then multiplied by it, gives 4956, the same as the dividend; therefore nothing remains, and 4·16 is the root of 17·3056, as required.

## Ex. II.

*What is the root of 2025?*

$$\begin{array}{r} 20,25(45 \text{ root} \\ 16 \\ \hline 85 \overline{) 425} \\ 5 \overline{) 425} \\ \hline \end{array}$$

## Ex. III.

*What is the root of .000729?*

$$\begin{array}{r} .00,07,29(.027 \text{ root} \\ 4 \\ \hline 47 \overline{) 329} \\ 7 \overline{) 329} \\ \hline \end{array}$$

*Note.* When all the periods of the given number are brought down and used, and more periods are required to be found, the operation may be continued by adding as many periods of ciphers as we please, namely, bringing always two ciphers at once to each dividend. And when the root is to be extracted to a great number of places, the work may be much abbreviated: thus, having proceeded in the extraction after the common method, till you have found one more than half the required number of figures in the root, the rest may be found by dividing the last remainder by its

corresponding divisor, annexing a cipher to every dividend, as in division of decimals; or rather, without annexing ciphers, by omitting continually the right hand figure of the divisor, after the manner of contracted division of decimals.

So the operation for the root 2, to 12 or 13 places, may be thus.

## EXAMPLE IV.

$$\begin{array}{r}
 2)1\cdot414213562373 \text{ root} \\
 \underline{1} \\
 24 \overline{)100} \\
 \underline{4} \phantom{0} 96 \\
 \hline
 281 \overline{)400} \\
 \underline{1} \phantom{0} 281 \\
 \hline
 2824 \overline{)11900} \\
 \underline{4} \phantom{0} 11296 \\
 \hline
 28282 \overline{)60400} \\
 \underline{2} \phantom{0} 56564 \\
 \hline
 282841 \overline{)383600} \\
 \underline{1} \phantom{0} 282841 \\
 \hline
 2828423 \overline{)10075900} \\
 \underline{3} \phantom{0} 8485269 \\
 \hline
 2828426) 1590631(562373 \\
 \dots\dots 176481 \\
 \phantom{000000} 6712 \\
 \phantom{000000} 1055 \\
 \phantom{000000} 206 \\
 \phantom{000000} 8
 \end{array}$$

Here, having found the first seven figures 1·414213 by the common extraction, by adding always periods of ciphers, the

the last six figures 562373 are found by the method of contracted division in decimals, without adding ciphers to the remainder, but only pointing off a figure at each time from the last divisor.

The use of the square root will be shown in Mensuration, where it will be more particularly wanted.

## PROBLEM XI.

To extract the cube root.

I. Point the given number into periods of three places each, beginning at units; and there will be as many integral places in the root, as there are points over the integers in the given number.

II. Seek the greatest cube in the left hand period; write the root in the quotient, and the cube under the first period; from which subtract it, and to the remainder bring down the next period: call this the resolvend, under which draw a line.

III. Under the resolvend, write the triple square of the root, so that units in the latter stand under the place of hundreds in the former; under the triple square of the root, write the triple root, removed one place to the right; and the sum of these two lines call a divisor, under which draw a line.

IV. Seek how often this divisor may be had in the resolvend, its right hand place excepted, and write the result in the quotient.

V. Under the divisor, write the product of the triple square of the root by the last quotient figure, setting the units place of this line under that of tens in the divisor; under this line, write the product of the triple root by the square of the last quotient figure; let this line be removed one place be-

yond the right of the former; and under this line, removed one place forward to the right, set the cube of the last quotient figure. The sum of these three lines call the subtrahend, under which draw a line.

VI. Subtract the subtrahend from the resolvend; to the remainder bring down the next period for a new resolvend; the divisor to this, must be the triple square of all the quotient added to the triple thereof, as in the third article, &c.

### EXAMPLE I.

*What is the cube root of 48228544?*

48228544(364	
27	
-----	
	21228      Resolvend
-----	
add	<div style="display: inline-block; vertical-align: middle;"> <math>\left\{ \begin{array}{l} 27 \\ 09 \end{array} \right.</math> </div> <div style="display: inline-block; vertical-align: middle; margin-left: 10px;"> <math>\left. \begin{array}{l} \text{Triple square of 3} \\ \text{Triple of 3} \end{array} \right\} \text{the root.}</math> </div>
-----	
	279      Divisor
-----	
add	<div style="display: inline-block; vertical-align: middle;"> <math>\left\{ \begin{array}{l} 162 \\ 324 \\ 216 \end{array} \right.</math> </div> <div style="display: inline-block; vertical-align: middle; margin-left: 10px;"> <math>\left. \begin{array}{l} \text{Triple square of 3 multiplied by 6} \\ \text{Triple of 3 multiplied by square of 6} \\ \text{Cube of 6} \end{array} \right.</math> </div>
-----	
	19656      Subtrahend
-----	
	1572544      Resolvend
-----	
add	<div style="display: inline-block; vertical-align: middle;"> <math>\left\{ \begin{array}{l} 3888 \\ 108 \end{array} \right.</math> </div> <div style="display: inline-block; vertical-align: middle; margin-left: 10px;"> <math>\left. \begin{array}{l} \text{Triple square of 36} \\ \text{Triple of 36} \end{array} \right\} \text{the root.}</math> </div>
-----	
	38988      Divisor
-----	
add	<div style="display: inline-block; vertical-align: middle;"> <math>\left\{ \begin{array}{l} 15552 \\ 1728 \\ 64 \end{array} \right.</math> </div> <div style="display: inline-block; vertical-align: middle; margin-left: 10px;"> <math>\left. \begin{array}{l} \text{Triple square of 36 multiplied by 4} \\ \text{Triple of 36 multiplied by square of 4} \\ \text{Cube of 4} \end{array} \right.</math> </div>
-----	
	1572544      Subtrahend
-----	

If

If the work of this Example be well considered, and compared with the foregoing rule, it will be easy to conceive how any other example of the like nature may be wrought. And here observe, that when the cube root is extracted to more than two places, there is a necessity of doing some work on a spare piece of paper, in order to come at the root's triple square, and the product of the triple root, by the square of the quotient figure, &c.

In this Example, the given number is a cube number, and therefore at the end of the operation there remained nothing; for 364 multiplied by 364, and the product multiplied by 364 again, gives 48228544, the given number.

But if the number given be not a cube number, then to the last remainder always bring down three ciphers, and work anew for a decimal fraction, if needful.

#### MORE EXAMPLES.

*What is the cube root of*

$$\begin{array}{r}
 389017 \\
 1092727 \\
 27054036008 \\
 219365327791 \\
 122615327232
 \end{array}
 \left. \vphantom{\begin{array}{r} 389017 \\ 1092727 \\ 27054036008 \\ 219365327791 \\ 122615327232 \end{array}} \right\} \text{Answers.} \left\{ \begin{array}{r} 73 \\ 103 \\ 3002 \\ 6031 \\ 4968 \end{array} \right.$$

These Examples are all performed in the same manner as the foregoing one.

The use of the cube root will be shown in Mensuration.



10

THE  
PRACTICE  
OF  
***MENSURATION,***  
SHEWING THE USES  
OF THE  
MOST PRINCIPAL RULES  
Applied to Measuring the  
SEVERAL ARTIFICER'S WORKS,  
CONCERNED IN  
**BUILDING.**



\*\*\*\*\*

## MENSURATION

OF

### SUPERFICIES.

#### DEFINITION.

**E**VERY quantity is measured by some other quantity of the same kind; as a line by a line, a surface by a surface, and a solid by a solid: and the number which shows how often the lesser, called the measuring unit, is contained in the greater, or quantity measured, is called the content of the quantity so measured. Thus, if the quantity to be measured be the rectangle ABCD, and the little square E, whose side is one inch, be the measuring unit propounded, then, as often as the said little square is contained in the rectangle, so many square inches the rectangle is said to contain: so that if the length DC be supposed 5 inches, and the breadth AD 3 inches, the content of the rectangle will be 3 times 5, or 15 square inches: because, if lines be drawn parallel to the sides, at an inch distance one from another, they will divide the whole rectangle ABCD into 3 times 5, or 15 equal parts, of one inch each. And, generally, whatever the measures of the two sides may be, it is evident that the rect-

VOL. I.

X

angle

angle will contain the square E, as many times as the base AB contains the base of the square, repeated as often as the altitude AD contains the altitude of the square.

Hence we have the following rule for any parallelogram whatever.

### PROBLEM I.

To find the area of a parallelogram, whether it be a square, a rectangle, a rhombus, or a rhomboides.

Multiply the length by the perpendicular height, and the product will be the area.

### EXAMPLE I.

*What is the area of a square ABCD, whose side AB or BC is 2f. 3i.?*

By duodecimals.

$$\begin{array}{r}
 2 \ 3i \\
 2 \ 3i \\
 \hline
 6 \ 9 \\
 4 \ 6 \\
 \hline
 \text{Ans. } 5 \ 0 \ 9 \\
 \text{ft. } i \ ii
 \end{array}$$

By decimals.

$$\begin{array}{r}
 2.25 \\
 2.25 \\
 \hline
 1125 \\
 450 \\
 \hline
 450 \\
 \hline
 \text{f. } 5.0625
 \end{array}$$

EX-

EXAMPLE II.

*What is the area of a rectangle ABCD, whose length AB is 12f. 6i. and the breadth BC 2f. 9i.*

By duodecimals.

$$\begin{array}{r}
 12 \text{ } 6i \\
 2 \text{ } 9i \\
 \hline
 9 \text{ } 4 \text{ } 6 \\
 25 \text{ } 0 \\
 \hline
 \text{Ans. } 34 \text{ } 4 \text{ } 6 \\
 \text{f. } i \text{ } ii
 \end{array}$$

By decimals.

$$\begin{array}{r}
 12.5 \\
 2.75 \\
 \hline
 625 \\
 875 \\
 250 \\
 \hline
 \text{f. } 34.375
 \end{array}$$

EXAMPLE III.

*What is the area of a rhombus ABCD, the length AB being 12f. 6i. and the height 6f. 3i.?*

By duodecimals

$$\begin{array}{r}
 12 \text{ } 6 \\
 6 \text{ } 3 \\
 \hline
 3 \text{ } 1 \text{ } 6 \\
 75 \text{ } 0 \\
 \hline
 \text{Ans. } 78 \text{ } 1 \text{ } 6 \\
 \text{f. } i \text{ } ii
 \end{array}$$

By decimals.

$$\begin{array}{r}
 12.5 \\
 6.25 \\
 \hline
 625 \\
 250 \\
 750 \\
 \hline
 \text{f. } 78.125
 \end{array}$$

## EXAMPLE. IV.

*What is the area of a rhomboides ABCD, whose length AB is 16f. 3i. and the height DE 5f. 6i.?*

By duodecimals.	By practice.	By decimals.
$\begin{array}{r} 16\ 3 \\ 5\ 6 \\ \hline 8\ 1\ 6 \\ 81\ 3 \\ \hline \text{Ans. } 89\ 4\ 6 \\ \text{f. } i\ ii \end{array}$	$\begin{array}{r} 16\ 3 \\ 5\ 6 \\ \hline 81\ 3 \\ 8\ 1\ 6 \\ \hline 89\ 4\ 6 \\ \text{f. } i\ ii \end{array}$	$\begin{array}{r} 16\cdot25 \\ 5\cdot5 \\ \hline 8125 \\ 3125 \\ \hline \text{f. } 89\cdot375 \end{array}$

## PROBLEM II.

**To find the area of a triangle.**

Multiply the base by the perpendicular height, and half the product will be the area.

## EXAMPLE I.

*What is the area of a triangle ABC, the base AB being 12f. 3i. and the height BC 8f. 6i.?*

By duodecimals.	By practice.	By decimals.
$\begin{array}{r} 12\ 3 \\ 8\ 6 \\ \hline 6\ 1\ 6 \\ 98\ 0 \\ \hline 2)104\ 1\ 6 \\ \hline \text{Ans. } 52\ 0\ 9 \\ \text{f. } i\ ii \end{array}$	$\begin{array}{r} 12\ 3 \\ 8\ 6 \\ \hline 98\ 0 \\ 6\ 1\ 6 \\ \hline 2)104\ 1\ 6 \\ \hline 52\ 0\ 9 \\ \text{f. } i\ ii \end{array}$	$\begin{array}{r} 12\cdot25 \\ 8\cdot5 \\ \hline 6125 \\ 9800 \\ \hline 2)104125 \\ \hline \text{f. } 52\cdot0625 \end{array}$

EX-

EXAMPLE II.

*What is the area of a triangle ABC, whose base AB is 18f. 4i. and the height CD, 10f. 3i.?*

By duodecimals.

$$\begin{array}{r}
 18 \ 4 \\
 10 \ 3 \\
 \hline
 4 \ 7 \ 0 \\
 183 \ 4 \\
 \hline
 2)187 \ 11 \ 0 \\
 \hline
 \text{Ans. } 93 \ 11 \ 6 \\
 \text{f. } i \ ii
 \end{array}$$

By practice.

$$\begin{array}{r}
 18 \ 4 \\
 10 \ 3 \\
 \hline
 183 \ 4 \\
 4 \ 7 \\
 \hline
 2)187 \ 11 \\
 \hline
 \text{Ans. } 93 \ 11 \ 6 \\
 \text{f. } i \ ii
 \end{array}$$

PROBLEM III.

To find the area of a triangle, whose three sides only are given.

From the half sum of the three sides, subtract each side severally; multiply the half sum and the three remainders together, and the square root of the product will be the area required.

EX-

## EXAMPLE I.

*Requireth the area of a triangle ABC, whose three sides AB, BC, and CA, are respectively 13, 14, and 15 feet.*

	13 14 15 — 2)42 —	
	21 the half sum of the sides	
21 13 —	21 14 —	21 15 —
8 first difference	7 second diff.	6 third difference.

21 ×8 — 168 ×7 — 1176 ×6 — 7056(82·77 feet the answer 64 — 162) 656 324 — 1647)33200 11529 — 16547) 216710 115829 — 100881
---

PROBLEM

# PROBLEM IV.

Any two sides of a right angled triangle being given, to find a third side.

## CASE I.

*When the two sides are given, to find the hypotenuse.*

Add the squares of the two legs together, and the square root of the sum will be the hypotenuse.

## EXAMPLE I.

*Requireth the hypotenuse AC of the right angle ABC, the side AB being 4, and the perpendicular BC 3.*

$$\begin{array}{rcl}
 4 \times 4 & = & 16 \text{ the square of AB.} \\
 3 \times 3 & = & 9 \text{ the square of BC.} \\
 \hline
 & & 25 \text{ (5 the answer.} \\
 & & 25 \\
 \hline
 \end{array}$$

## EXAMPLE II.

*In the right angle triangle ABC, the base AB is 56, and the perpendicular BC is 33, requireth the hypotenuse.*

$  \begin{array}{r}  56 \\  56 \\  \hline  336 \\  280 \\  \hline  3136 \text{ square} \\  \hline  \end{array}  $	$  \begin{array}{r}  33 \\  33 \\  \hline  99 \\  99 \\  \hline  1089 \\  3136 \\  \hline  4225(65 \\  36 \\  \hline  125)625 \\  625 \\  \hline  \end{array}  $
---	--

EX-

## EXAMPLE III.

*There is a roof, whose span AB is 30 feet, and its height CD 12 feet; what is the length of a rafter, as AD or BD?*

This is only two right angled triangles of one continued base, joined together at their perpendicular.

2)30	12	
<hr style="width: 50px; border: 0.5px solid black;"/>	12	
15	<hr style="width: 50px; border: 0.5px solid black;"/>	
15	144	
<hr style="width: 50px; border: 0.5px solid black;"/>	+225	
75	<hr style="width: 50px; border: 0.5px solid black;"/>	
15	369(19.209× =AD or DB the ans.	
<hr style="width: 50px; border: 0.5px solid black;"/>	1	
225	<hr style="width: 50px; border: 0.5px solid black;"/>	
	29)269	
	261	
	<hr style="width: 50px; border: 0.5px solid black;"/>	
	382) 800	
	764	
	<hr style="width: 50px; border: 0.5px solid black;"/>	
	38409) 360000	
	345681	
	<hr style="width: 50px; border: 0.5px solid black;"/>	
	14319	

EX-

EXAMPLE IV.

*The span or width of a roof is 48 feet, and the height 18 feet, standing upon a rectangle plan hiped at each end, I demand the length of the common rafters, and likewise the hip rafters.*

2)48	18
24 the length of the base	18
24	144
96	18
48	324
+576	
+324	
900(30 the length of each common rafter.	
9	
60) 00	
00	

But because the hip rafters are the hypotenuses of right-angled triangles, having a common rafter for one of the perpendicular legs, and the other leg being equal to half the width of the roof.

Hence  $30 \times 30 = 900$   
 And  $24 \times 24 = 576$

1476(38'41 feet the length of a hip rafter.
9
68) 576
544
764) 3200
3056
7681) 14400
7681
6719

## CASE II.

*The hypotenuse and one of the legs being given, to find the other leg.*

From the square of the hypotenuse, take the square of the given leg, and the square root of the remainder will be equal to the other leg.

## EXAMPLE I.

*In the right-angled triangle ABC, the base AB being 40, and the hypotenuse AC 50, I demand the perpendicular BC.*

$$\begin{array}{r}
 50 \times 50 = 2500 \\
 40 \times 40 = 1600 \\
 \hline
 900 (30 = BC \\
 9 \\
 \hline
 60) 00 \\
 00 \\
 \hline
 \end{array}$$

## EXAMPLE II.

*The width of a roof being 36 feet, and the length of a rafter 25 feet, I demand the height of the roof.*

$$\begin{array}{r}
 25 \\
 25 \\
 \hline
 125 \\
 50 \\
 \hline
 625 \\
 324 \\
 \hline
 301 (17.3 \text{ the height nearly.} \\
 1 \\
 \hline
 27) 201 \\
 189 \\
 \hline
 343) 1200 \\
 1029 \\
 \hline
 17100
 \end{array}
 \qquad
 \begin{array}{r}
 2) 36 \\
 \hline
 18 \text{ the base.} \\
 18 \\
 \hline
 144 \\
 18 \\
 \hline
 324
 \end{array}$$

PROBLEM

## PROBLEM V.

To find the area of a trapezium.

Multiply the diagonal by half the sum of the two perpendiculars, falling upon it from the opposite angles, and the product will be the area.

### EXAMPLE I.

*What is the area of a trapezium ABCD, the diagonal AC being 36 feet, the perpendicular DE 16 feet, and BF 12 feet?*

$$\begin{array}{r}
 16 \\
 +12 \\
 \hline
 28 \text{ the sum of DE and BF.} \\
 \hline
 14 \\
 \times 36 \\
 \hline
 84 \\
 42 \\
 \hline
 504 = \text{the area of ABCD.}
 \end{array}$$

## PROBLEM VI.

To find the area of a trapezoid.

Multiply the half sum of the parallel sides by the perpendicular distance between them, and the product will be the area.

## EXAMPLE I.

*What is the area of a board or plank in the form of a trapeziod, being 1f. 7i. one end, 2f. 3i. at the other end, and 8f. 6i. long;*

$$\begin{array}{r}
 1 \ 7 \\
 2 \ 3 \\
 \hline
 2)3 \ 10 \\
 \hline
 1 \ 11 \\
 8 \ 6 \\
 \hline
 15 \ 4 \\
 \phantom{15} 11 \ 6 \\
 \hline
 16 \ 3 \ 6 \\
 \text{f.} \quad \text{i} \quad \text{ii}
 \end{array}$$

## PROBLEM VII.

**To find the area of any regular polygon.**

Multiply half the perimeter of the figure by the perpendicular, falling from its centre upon one of the sides, and the product will be the area of the polygon.

## EXAMPLE I.

*Requireth the area of a regular pentagon ABCD, whose side AB, or BC, &c. is 6 feet, and the perpendicular EF 4f.*

$$\begin{array}{r}
 6 \\
 5 \\
 \hline
 2)30 \\
 \hline
 15 \text{ half the perimeter.} \\
 4 \\
 \hline
 60 \text{ feet, equal the area required.}
 \end{array}$$

EX-

EXAMPLE II.

*How many feet of ground does an hexangular building cover, each side of the base being 8f. 3i. and the perpendicular 7 feet?*

$$\begin{array}{r}
 8 \ 3 \\
 6 \\
 \hline
 2)49 \ 6 \\
 \hline
 24 \ 9 \text{ half the sum of the sides.} \\
 7 \\
 \hline
 f.173 \ 3
 \end{array}$$

PROBLEM VIII.

To find the area of a polygon, when the side only is given.

Multiply the square of the given side of the polygon by that number which stands opposite to its name in the following table, and the product will be the area.

No. of sides.	Names.	Multiplier.
3	Trigon or equilateral triang.	0·43301+
4	Tetragon or square - - -	1·00000+
5	Pentagon - - - - -	1·72047+
6	Hexagon - - - - -	2·59807+
7	Heptagon - - - - -	3·63391+
8	Octagon - - - - -	4·82842+
9	Nonagon - - - - -	6·18182+
10	Decagon - - - - -	7·69420—
11	Undecagon - - - - -	9·36564+
12	Duodecagon - - - - -	11·19615+

In the above Table, those multipliers marked with the sign +, are rather too small; on the contrary, those marked —, are too great: I have only given this Table to five places of decimals, being exact enough for most practical purposes.

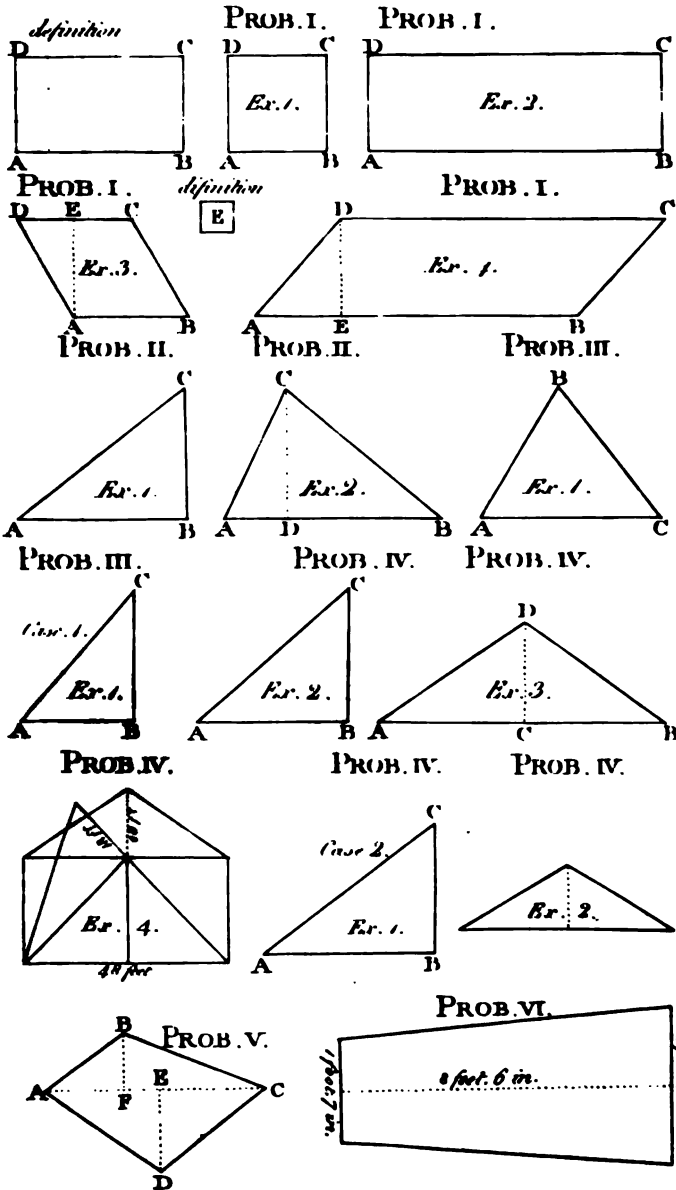
#### EXAMPLE I.

*Requireth the area of a pentagon, each side being 14 feet.*

14	1·72047
14	196
<hr/>	
56	1032282
14	1548423
<hr/>	
196	172047
<hr/>	
	337·21212 feet ans.
<hr/>	

#### PROBLEM

# MENSURATION OF SUPERFICIES





# PROBLEM IX.

The diameter of a circle being given, to find the circumference; or the circumference being given, to find the diameter.

## METHOD I.

As 7 is to 22, so is the diameter to the circumference nearly; or, as 22 is to 7, so is the circumference to the diameter nearly.

## EXAMPLE I.

*What is the circumference of a circle, whose diameter is 12 feet?*

$$\begin{array}{r}
 7 \text{ : } 22 \text{ :: } 12 \\
 \hline
 7 \overline{)264} \\
 \hline
 37 \text{ } 8 \\
 \text{f. } \text{i.}
 \end{array}$$

## EXAMPLE II.

*What is the diameter of a circle whose circumference is 63 feet?*

$$\begin{array}{r}
 22 \text{ : } 7 \text{ :: } 63 \\
 \hline
 22 \overline{)441} (20 \text{ } 0 \text{ } 6 \\
 \hline
 44 \\
 \hline
 1 \\
 12 \\
 \hline
 12 \\
 12 \\
 \hline
 22 \overline{)144} \\
 132 \\
 \hline
 12
 \end{array}$$

The

The most practical method to find the circumference of a circle from its diameter, is the following :

Multiply the diameter by 3 : add  $\frac{1}{4}$  part of the diameter to the product, the sum will be the circumference, the same as in Method I.

### EXAMPLE I.

*What is the circumference of a circle, whose diameter is 14f. 6i.?*

$$\begin{array}{r}
 14 \ 6 \\
 \times 3 \\
 \hline
 43 \ 6 \\
 2 \ 0 \ 10 \\
 \hline
 45 \ 6 \ 10 \\
 \text{f.} \quad \text{i.} \quad \text{ii.}
 \end{array}$$

### EXAMPLE II.

*What must be the length of a veneer to bend round a circular cylinder, whose diameter is 3f. 6i.?*

$$\begin{array}{r}
 3 \ 6 \\
 \times 3 \\
 \hline
 10 \ 6 \\
 0 \ 6 \\
 \hline
 \text{feet } 11 \ 0 \text{ ans.}
 \end{array}$$

EX-

EXAMPLE III.

*What is the circumference of a semicircular vault, whose diameter is 16f. 5in.?*

$$\begin{array}{r}
 \text{f. in.} \\
 16 \quad 5 \\
 \times 3\frac{1}{2} \\
 \hline
 49 \quad 3 \\
 2 \quad 4 \quad 1 \\
 \hline
 2)51 \quad 7 \quad 1 \\
 \hline
 \text{feet } 25 \quad 9 \quad 6 \text{ answer nearly.} \\
 \hline
 \end{array}$$

METHOD II.

Multiply the diameter by 3·1416, and the product will be the circumference.

Or divide the circumference by 3·1416, and the quotient will be the diameter.

This method comes nearer to the truth than the foregoing, but for practical purposes, Method I. will be sufficiently near.

PROBLEM X.

The chord and height of a segment being given, to find the chord of half the arc.

To the square of the half chord, add the square of the height, and the square root of the sum will be the length of the chord of half the arc.

## EXAMPLE.

*The chord AC being 48 feet, and the height DB 18 feet, what is the length of the chord of half the arc?*

2)48		18
<hr/> 24		<hr/> 18
24		144
<hr/> 96		<hr/> 18
48		324
<hr/>		

add { 576 square of half the chord  
324 square of the height

---

900 (30 length of half the chord.  
9

---

00

## PROBLEM XI.

To find the length of any arc of a circle, the half chord and chord of the whole arc being given.

Subtract the chord of the whole arc from double the chord of the half arc: add  $\frac{1}{3}$  of the remainder to the double chord of the half arc, and the sum will be nearly equal to the length of the arc.

## EXAMPLE I.

*What is the length of the arc ACB, whose chord AB is 48, and the half chord AC is 30?*

$2 \times 30 = 60$  the double chord of the half arc.  
 $- 48$  the chord of the whole arc.  


---

3)12  


---

4  


---

64 the length of the arc required.

## PROBLEM

## PROBLEM XII.

The chord and height of a segment being given,  
to find the radius of the circle.

To the square of the half chord, add the square of the height, and divide the sum by twice the height of the segment, and the quotient will be the radius of the circle, when it is less than a semicircle.

### EXAMPLE.

*The chord AC of a segment ABC being 48 feet, and the height 18 feet, what is the radius of the circle?*

2)48		18 the height
24 half the chord		18
24		144
96		18
48		324
576	square of the half chord	
324	the square of the height	
900	25 the radius required	
72		18
180		2
180		36 twice the height

## PROBLEM XIII.

Given any two parallel chords in a circle, and their distance, to find the distance of the greater chord from the centre.

To the square of the distance between the chords, add the square of half the lesser chord. The difference between this sum, and the square of half the greater chord divided by twice the distance of the chords, will give the distance of the centre from the greatest chord.

## EXAMPLE.

Suppose the greater chord CD is 48 feet, and the lesser AB 30, their distance EG 13 feet, what is the distance EF from the centre to the greater chord CD?

13	$\frac{3^2}{2} = 15$	$\frac{4^2}{2} = 24$	
13	15	24	
39	75	96	
13	15	48	
169	225	576	[chord
	sq. of the less.	sq. of the greater ch.	
	169	—394	sq. of the dist.
	394	182	
		2 × 13 = 26) 182 = EF dist. required	
		182	

PROBLEM

# PROBLEM XIV.

Given a chord of a circle and its distance from the centre, to find the radius of the circle.

To the square of the half chord, add the square of the distance from the centre, and the square root of the sum will be the radius required.

## EXAMPLE.

Given the chord CD 48 feet, and its distance EF from the centre 7 feet, required the radius of the circle.

$$\begin{array}{rcl}
 \frac{48}{2} = 24 & \text{and} & 24 \times 24 = 576 \\
 7 \times 7 = 49 & & \\
 \hline
 & & 625 \text{ (25 the radius)} \\
 & & \sqrt{\phantom{00}} \\
 45 \overline{) 225} & & \\
 \underline{225} & & 
 \end{array}$$

# PROBLEM XV.

Given any two parallel chords in a circle, and the distance between them, to find the perpendicular height from the middle of either chord to the circumference.

Find the nearest distance of the greater chord from the centre, by Problem XIII, and find the radius of the circle by

by Problem XIV. add the distance between the two parallel chords, and the distance between the greater chord, and the centre of the circle together: this sum being taken from the radius, will give the perpendicular height from the middle of the lesser chord, to the circumference or height of the lesser segment; to the lesser segment, add the distance between the parallel chords, and the sum will be the height of the greater segment.

### EXAMPLE.

*Given the greater chord CD 48 feet, and the lesser chord AB 30 feet, their distance EG 13 feet, required the distance GH perpendicular from the middle of AB to the circumference.*

The distance from the centre to the greater chord, will be found to be 7 feet, by Problem XIII. and the radius 25 feet, by Problem XIV.

$13 + 7 = 20$  and  $25 - 20 = 5$  feet, height of the lesser seg.

Then  $13 + 5 = 18$  the height of the greater segment.

### PROBLEM XVI.

To find the area of a circle, the diameter being given.

#### METHOD I.

Multiply half the circumference by half the diameter, and the product will be the area.

EX-

EXAMPLE I.

*What is the area of a circle whose diameter is 28 feet, and its circumference 88 feet?*

$$\begin{array}{r}
 \frac{88}{2} = 44 \\
 \frac{28}{2} = 14 \\
 \hline
 176 \\
 4 \\
 \hline
 616 \text{ feet, the area required.}
 \end{array}$$

METHOD II.

Multiply the square of the diameter by .7854, and the product will be the area.

EXAMPLE I.

*What is the area of a circle whose diameter is 3f. 6i.*

$$\begin{array}{r}
 3 \cdot 5 \\
 3 \cdot 5 \\
 \hline
 175 \\
 105 \\
 \hline
 12 \cdot 25 \text{ square of the diameter.} \\
 7854 \\
 \hline
 4900 \\
 6125 \\
 9800 \\
 8575 \\
 \hline
 9 \cdot 621150 \text{ the answer.}
 \end{array}$$

In common practice, multiply the square of the diameter when given in feet, inches, &c. by 9i. 5ii.

EX-

## EXAMPLE.

*What is the area of a circle whose diameter is 3f. 6i.?*

$$\begin{array}{r}
 \begin{array}{r}
 \text{f.} \quad \text{i.} \\
 3 \quad 6 \\
 3 \quad 6\text{i.} \\
 \hline
 1 \quad 9 \quad 0 \\
 10 \quad 6 \\
 \hline
 12 \quad 3\text{i} \\
 9 \quad 5\text{ii} \\
 \hline
 5 \quad 1 \quad 3\text{iii} \\
 9 \quad 2 \quad 3 \\
 \hline
 9 \quad 7 \quad 4 \quad 3 \text{ the answer.} \\
 \text{f.} \quad \text{i} \quad \text{ii} \quad \text{iii}
 \end{array}
 \end{array}$$

## METHOD III.

When the circumference is given.

Multiply the square of the circumference by .07958, and the product will be the area

## EXAMPLE.

*What is the area of a circle when the circumference is 88 feet?*

$$\begin{array}{r}
 88 \\
 88 \\
 \hline
 704 \\
 704 \\
 \hline
 7744 \text{ square of the circumference.} \\
 \cdot 07958 \\
 \hline
 61952 \\
 38720 \\
 69696 \\
 54208 \\
 \hline
 61626752 \text{ the area of the circle.} \\
 \hline
 \hline
 \end{array}$$

## PROBLEM

## PROBLEM XVII.

To find the area of a sector of a circle.

Multiply the radius, or half the diameter, by half the length of the arc of the sector, and the product will be the area.

### EXAMPLE I.

*What is the area of a sector ABC, the arc BC being 3f. 6i. and the radius AB or AC 6f. 2i.?*

$$\begin{array}{r}
 \begin{array}{r}
 \text{f.} \quad \text{i.} \\
 2)3 \quad 6 \\
 \hline
 1 \quad 3 \\
 6 \quad 2 \\
 \hline
 0 \quad 3 \quad 6\text{ii} \\
 10 \quad 6 \\
 \hline
 10 \quad 9 \quad 6 \\
 \text{f.} \quad \text{i.} \quad \text{ii.}
 \end{array}
 \end{array}$$

## PROBLEM XVIII.

To find the area of the segment of a circle, the chord and height of the arc being given.

Find the length of the arc ABC by Prob. XI. and the radius of the circle by Prob. XII. the area of the sector ABCE by Prob. XVII.

Subtract the area of the triangle AEC found by Prob. II. from the area of the sector, and the remainder will be the area of the segment.

## EXAMPLE.

*What is the area of the segment of a circle ABC, the chord AC being 48 feet, and the height DB 18 feet?*

The length of the arc will be found to be 64 feet, and the radius 25 feet, then

2)64	25
<hr/>	<hr/>
32	—18
<hr/>	<hr/>
×25	7 perpendicular DE
<hr/>	×48
160	<hr/>
64	2)336
<hr/>	<hr/>
800 area of the sector	168 area of the triang. ACE
168	
<hr/>	
632 area of the segment.	

## METHOD II.

Multiply the height by 0.626 to the square of the product; add the square of half the chord; multiply twice the square root of the sum by two thirds of the height, and the product is the area, nearly.

EX.

EXAMPLE.

*What is the area of a circular segment ABC, whose height is 18 feet, and the chord 48 feet ?*

$  \begin{array}{r}  .626 \\  \times 18 \\  \hline  5008 \\  626 \\  \hline  11.268 \\  \times 11.268 \\  \hline  90144 \\  67608 \\  22536 \\  11268 \\  11268 \\  \hline  \end{array}  $	$  \begin{array}{r}  24 \\  \times 24 \\  \hline  96 \\  48 \\  \hline  576 \text{ square of the half chord.} \\  \\  3)18 \\  \hline  6 \\  \times 2 \\  \hline  12 \text{ two thirds of the height.}  \end{array}  $
<p>add { 126.967824</p>	
<p>576</p>	
<p>702.967824)26.513 square root of the sum</p>	
<p>4 53.026=26.513 × 2</p>	
<p>× 12</p>	
<p>46)302 636.312 area of the segment</p>	
<p>276</p>	
<p>525) 2696</p>	
<p>2625</p>	
<p>5301) 7178</p>	
<p>5301</p>	
<p>53023)187724</p>	
<p>159069</p>	

METHOD 111.

To two thirds of the product of the base multiplied by the height, add the cube of the height divided by twice the length of the segment, and the sum will be nearly the area.

$$\frac{2}{3} \times \frac{b \times h}{2} + \frac{h^3}{2l}$$

EXAMPLE.

What is the area of a circular segment, the chord AB being 48 feet, and the height CD 18 feet?

$$\begin{array}{r} 48 \\ \times 18 \\ \hline 384 \\ 48 \\ \hline 3)864 \\ \hline 288 \\ \times 2 \\ \hline 576 \\ +6075 \\ \hline \end{array}$$

636.75 the area required

$$2 \times 48 = 96 \quad 5832 \div 96 = 60.75$$

$$\begin{array}{r} 576 \\ 720 \\ 672 \\ \hline 480 \\ 480 \\ \hline \end{array}$$

This method will be sufficiently near for all practical purposes, and is much shorter than the two first methods.

PROBLEM

## PROBLEM XIX.

To find the area of a circular zone, which is that part of a circle laying between two parallel chords, and the parts of the circle intercepted by the chords,

Find the height of each segment by Problem XV. and the diameter by Problem XIV.; then the difference of the segments found by Problem XVIII. will be the answer.

EX.

## EXAMPLE.

The greater chord CD of a circular zone being 48 feet, and the lesser chord AB 30 feet, their distance FG 13 feet, required the area of the zone.

The distance EF will be found by Problem XIII. to be 7 feet.

The radius EF will be found by Problem XIV. to be 25 feet.

$13+7=20$ , and  $25-20=5$  the height of the lesser segment.

$13+5=18$  the height of the greater segment.

$\begin{array}{r} 30 \\ \times 5 \\ \hline 3)150 \\ \hline 50 \\ \times 2 \\ \hline 100 \\ +2.0833 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline 25 \\ \times 5 \\ \hline 2 \times 30 = 6,0)12,5 \\ \hline 2.0833 \\ \hline \end{array}$
---	---

102.0833 the area of the lesser segment.

$\begin{array}{r} 48 \\ \times 18 \\ \hline 384 \\ 48 \\ \hline 3)864 \\ \hline 288 \\ \times 2 \\ \hline 576 \\ +60.75 \\ \hline 636.75 \text{ area of the greater segment.} \\ -102.0833 \text{ area of the lesser segment.} \\ \hline 534.6667 \text{ area of the zone.} \\ \hline \end{array}$	$\begin{array}{r} 18 \\ \times 18 \\ \hline 144 \\ 18 \\ \hline 324 \\ \times 18 \\ \hline 2592 \\ 324 \\ \hline 2 \times 48 = 96)5832(60.75 \\ 576 \\ \hline 720 \\ 672 \\ \hline 480 \\ 480 \\ \hline \end{array}$
--	--

PROBLEM

## PROBLEM XX.

To find the diameter of a circle whose area shall be in a given proportion to the area of a circle whose diameter is known.

If the area is required to be greater than the given circle, multiply the given diameter by the square root of the intended increase, and it will give the diameter of the circle required.

But if the area is intended to be less than the area of the given circle, divide the given diameter by the square root of the intended decrease, which will give the diameter of the given circle.

### EXAMPLE I.

*What is the diameter of a circle, whose area is 9 times as much as one of 21 inches diameter?*

$$\sqrt{9}=3, \text{ then } 21 \times 3=63 \text{ inches.}$$

### EXAMPLE II.

*What is the diameter of a circle, whose area is  $\frac{1}{9}$  of a circle of 21 inches diameter?*

$$\sqrt{9}=3, \text{ then } \frac{21}{3}=7.$$

PROB-

## PROBLEM XXI.

To find the circumference of an ellipsis, the transverse and conjugate axis being given.

Multiply half the sum of the two axes by 3; to the product add  $\frac{1}{4}$  part of the sum of the two axes, and this sum will give the circumference near enough for most practical purposes.

## EXAMPLE I.

*What is the circumference of an ellipsis, whose transverse axis is 24 feet, and the conjugate 18 feet?*

$$\begin{array}{r}
 24 \\
 +18 \\
 \hline
 2)42 \\
 \hline
 21 \\
 \times 3\frac{1}{4} \\
 \hline
 63 \\
 + 3 \\
 \hline
 \end{array}$$

66 feet the circumference.

## EXAMPLE II.

*The width of an elliptical vault being 21f. 7i. and the height 7f. 3 $\frac{1}{2}$ i. what is the circumference?*

$$\begin{array}{r}
 2)21 \ 7 \\
 \hline
 10 \ 9 \ 6 \\
 7 \ 3 \ 6 \\
 \hline
 18 \ 1 \\
 \times 3\frac{1}{4} \\
 \hline
 54 \ 3 \\
 +2 \ 7 \\
 \hline
 \end{array}$$

56 10 the circumference.  
ft. in.

## PROBLEM

## PROBLEM XXII.

To find the area of an ellipsis, the transverse and conjugate axes being given.

Multiply the transverse axis by the conjugate, and the product by .7854, will give the area required.

### EXAMPLE.

*What is the area of an ellipsis, whose transverse axis is 24 feet, and the conjugate 18 feet ?*

$\begin{array}{r} 24 \\ \times 18 \\ \hline 192 \\ 24 \phantom{0} \\ \hline 432 \end{array}$	$\begin{array}{r} .7854 \\ \times 432 \\ \hline 15708 \\ 23562 \\ 31416 \\ \hline 3392928 \text{ answer.} \end{array}$
--	--

## PROBLEM XXIII.

To find the area of a parabola, the base or double ordinate being given, and the axis or height.

Multiply the base by the height, and two thirds of this product will be the area required.

## EXAMPLE.

*What is the area of a parabola, the axis CD being 12, and the double ordinate AB 18?*

$$\begin{array}{r}
 12 \\
 \times 18 \\
 \hline
 3)216 \\
 \hline
 72 \\
 \times 2 \\
 \hline
 144 \text{ answer.} \\
 \hline
 \end{array}$$

## PROBLEM XXIV.

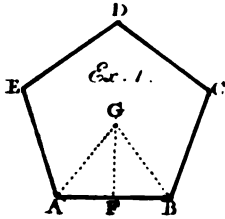
To find the area ABCD, of the frustum of a parabola, whose parallel ends AB and CD are given; also their distance EF.

To the square of the greatest end, add the square of the lesser end, to the product of the ends: divide the sum by the sum of the ends, and the quotient multiplied by the distance of the ends, two thirds of the product will be the answer.

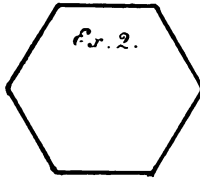
EX-

# MENSURATION OF SUPERFICIES

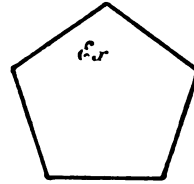
PROB. VII.



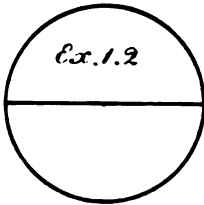
PROB. VII.



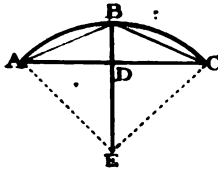
PROB. VIII.



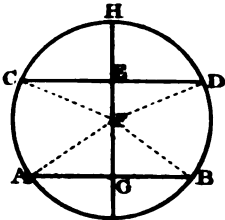
PROB. IX.



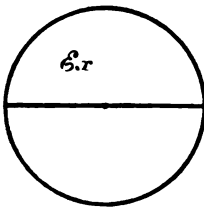
PROB. X. XLII



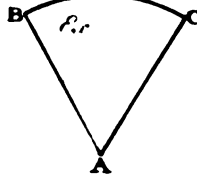
PROB. XIII. XLIV.



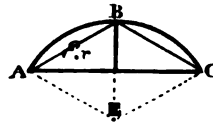
PROB. XVI



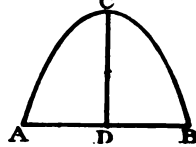
PROB. XVII



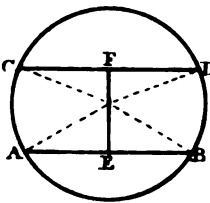
PROB. XVIII



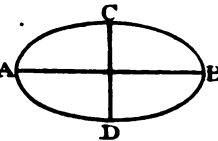
PROB. XXIII



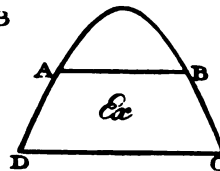
PROB. XIX



PROB. XXI. XXII



PROB. XXIV





EXAMPLE.

*Suppose the end DC 24, the end AB 20, and their distance EF 5, required the area ABCD.*

$  \begin{array}{r}  24 \\  \times 24 \\  \hline  96 \\  48 \phantom{0} \\  \hline  576  \end{array}  $	$  \begin{array}{r}  20 \\  \times 20 \\  \hline  400 \\  576 \\  \hline  480  \end{array}  $	$  \begin{array}{r}  24 \\  \times 20 \\  \hline  480  \end{array}  $
$  \begin{array}{r}  \hline  24 + 20 = 44 \quad 1456( \quad 33 \\  \phantom{24 + 20 = 44} 132 \phantom{00} 5 \\  \hline  136 \quad 3)165 \\  \phantom{136} 132 \phantom{00} \\  \hline  \phantom{136} 33 \phantom{00} 55 \\  \phantom{136} \phantom{33} 4 \phantom{00} 2 \\  \hline  \phantom{136} \phantom{33} \phantom{4} 110 \text{ answer nearly.}  \end{array}  $		

## MENSURATION

OF

## SOLIDS.

  
DEFINITION.

I. **AS** a line can only be measured by a line, and a surface by a surface, so a solid can only be measured by a solid; therefore solid measure is the finding the number of cubic inches, feet, yards, &c. contained in any thing that consists in length, breadth, and thickness.

II. A prism is a solid, whose ends are similar and equal parallel planes, and whose sides are parallelograms, and is denominated from the number of the sides of its base, as A, B, C, D.

III. If the ends and sides are equal squares, the solid is called a cube, as A.

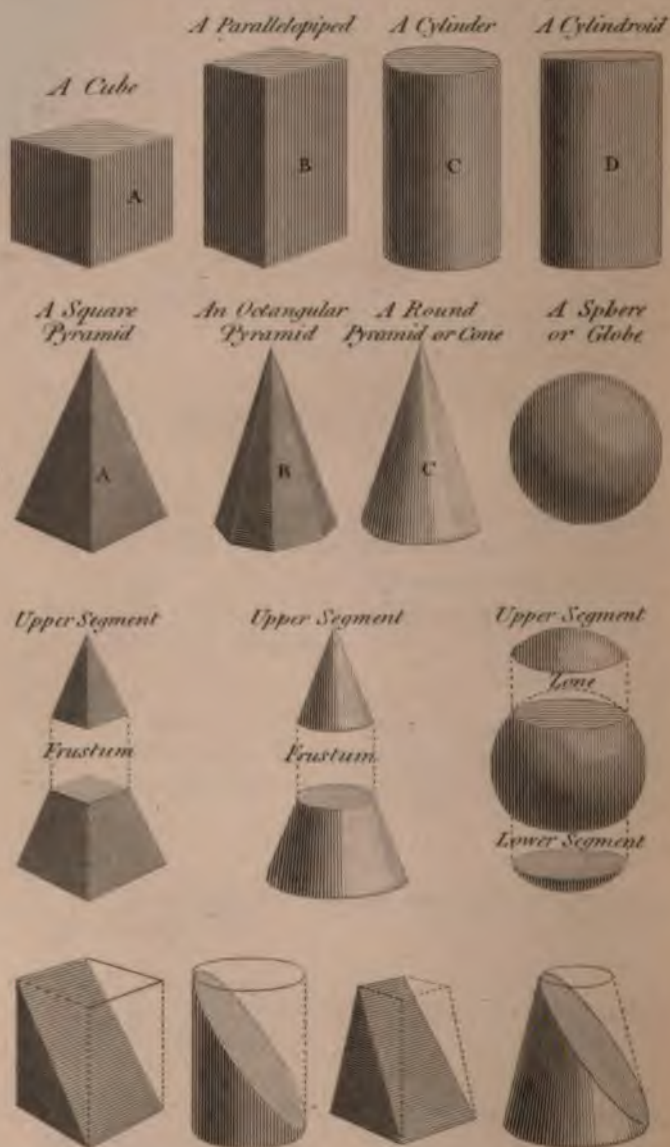
IV. If the base or ends are rectangles, the solid is called a parallelopiped, as B.

V. If the base or ends are circles, the solid is called a cylinder, as C.

VI. If



# MENSURATION of SOLIDS



VI. If the ends or bases are ellipses, the prism is called a cylindroid, as D.

VII. A solid standing upon any plane figure for its base, and whose sides are plane triangles meeting in one point, is called a pyramid, as A, B, and C.

VIII. If the base is a circle or an ellipse, then the pyramid is called a cone, as C.

IX. A solid which is terminated by a convex surface, which is everywhere equally distant from a certain point within, is called a sphere, or globe.

X. In a pyramid, cone, sphere, or any other tapering solid, a part thereof contained between two parallel ends, is called a frustum; and the parts wanting at the ends to complete the tapering solid, are called segments.

XI. If a frustum or any tapering solid be cut by a plane diagonally, from the extremity at one end, to the opposite extremity at the other, each of these pieces is called a hoof, or an ungula, that being the greatest which has the greatest base.

## PROBLEM I.

To find the area of a prism.

Multiply the area of the base or end, by the perpendicular height, and the product will give the solidity.

EX-

## EXAMPLE I.

*What is the solidity of a cube whose side is 12 inches?*

$12 \times 12 = 144$ , the area of the end, or 144 cubic inches, at one inch deep.

$144 \times 12 = 1728$ , the number of cubic inches in a foot.

## EXAMPLE II.

*What is the solidity of a prism, whose length AB is 10 feet, the breadth AC 5f. 9i. and the depth AD 3f. 6i.?*

	f.	i.	
	5	9	
	3	6i	
<hr/>			
2	10	6	
17	3		
<hr/>			
20	1	6	area of the base.
10			
<hr/>			
201	3	0	answer.
f.	i	ii	

EXAMPLE III.

*Required the area of a triangular prism, the height being 12f. 3in. one side of the base 3f. 6in. and the perpendicular of the triangle to that side 2f. 4in.*

	f.	i.	
	3	6	
	2	4	
<hr/>			
	1	2	0
	7	0	
<hr/>			
2)	8	2	0
<hr/>			
	4	1	
	12	3	
<hr/>			
	1	0	3
	49	0	
<hr/>			
50	0	3	solid contents of the whole.
f.	i.	ii.	

EXAMPLE IV.

*What is the solidity of a cylinder, whose height is 12 feet, and the diameter of the base 2·5 feet?*

2·5	
2·5	
<hr/>	
125	
50	
<hr/>	
6·25	
78 54	
<hr/>	
2500	
3125	
5000	
4375	
<hr/>	
4·908750	area of the base
12	
<hr/>	
58·905000	
<hr/>	

EX-

## EXAMPLE V.

*What is the solidity of a cylinder, when the circumference of the base is 7·85 feet, and the height 12 feet?*

$$\begin{array}{r}
 7\cdot85 \\
 \times 7\cdot85 \\
 \hline
 3925 \\
 6280 \\
 5495 \\
 \hline
 61\cdot6225 \\
 \times 0\cdot7958 \\
 \hline
 4929800 \\
 3081125 \\
 5546025 \\
 4313575 \\
 \hline
 4\cdot903918550 \text{ area of the base.} \\
 12 \\
 \hline
 58\cdot847022600 \text{ solidity of the cylinder.} \\
 \hline
 \end{array}$$

## PROBLEM II.

To find the solidity of a pyramid.

Multiply the area of the base or end by the perpendicular height, and one third of the product will give the solidity.

EX-

EXAMPLE I.

*What is the solidity of a pyramid, whose height is 9f. 6in. and each side of the base 2f. 3in.?*

$$\begin{array}{r}
 \text{f. in.} \\
 2 \quad 3 \\
 \times 2 \quad 3 \\
 \hline
 6 \quad 9 \\
 4 \quad 6 \\
 \hline
 5 \quad 0 \text{ i.} \quad 9 \text{ ii.} \quad \text{area of the base.} \\
 \times 9 \quad 6 \text{ i.} \\
 \hline
 2 \quad 6 \quad 4 \quad 6 \\
 45 \quad 6 \quad 9 \\
 \hline
 3)48 \quad 1 \quad 1 \quad 6 \\
 \hline
 16 \quad 0 \quad 4 \quad 6 \quad \text{solid content of pyramid.} \\
 \text{f. i. ii. iii.}
 \end{array}$$

EXAMPLE II.

*Required the solidity of a cone, the diameter of the base being 2f. 6in. and the height 12f.*

$$\begin{array}{l}
 \text{f. in.} \\
 2 \quad 6 = 2.5 \\
 2.5 \times 2.5 \times .7854 \times 12 = 58.905 \text{ the solidity of a} \\
 \text{cylinder of the same base and altitude.}
 \end{array}$$

$$\text{And } \frac{58.905}{3} = 19.635 \text{ the solidity of the cone.}$$

## PROBLEM III.

To measure the frustum of a square pyramid.

To the rectangle of the sides of the two ends, add the sum of their squares; that sum being multiplied by the height, one third of the product will give the solidity.

## EXAMPLE.

Let ABCDEFG be the frustum of a square pyramid, one side of the base AB, or BC, being 3f. 6in.; each side DE, or EF, of the top 2f. 3in.; and the perpendicular height HI, 6f. 9in.

	f.	in.	
	3	6	= 3.5
	2	3	= 2.25
	6	9	= 6.75
3.5	2.25	2.25	side of the lesser base.
× 3.5	× 2.25	× 3.5	side of the greater base.
175	1125	1125	
105	450	675	
12.25	450		
	5.0625		
		7.875	rectangle of the two bases.
		5.0625	square of the top end.
		12.25	square of the base.
		25.1875	sum.
		× 6.75	height.
		1259375	
		1763125	
		1511250	
		3)170.015625	
		56.671875	solidity of the frustum.

METHOD

METHOD II.

To the rectangle of the sides of the two bases, add one third of the square of their difference; that sum being multiplied by the height, will give the solidity.

EXAMPLE.

*Let ABCDEFG be the frustum of a square pyramid, one side of the base AB, or BC, being 3f. 6in.; each side DE, or EF, of the top, being 2f. 3in.; and the perpendicular height HI, 6f. 9in.; required the solidity.*

$  \begin{array}{r}  \text{f. in.} \\  3 \quad 6 \\  \times 2 \quad 3 \\  \hline  10 \quad 6 \\  7 \quad 0 \\  \hline  7 \quad 10 \quad 6 \text{ rectangle of the sides.} \\  + 0 \quad 6 \quad 3 \\  \hline  8 \quad 4 \quad 9 \\  \times 6 \quad 9 \\  \hline  6 \quad 3 \quad 6 \quad 9 \\  50 \quad 4 \quad 6 \\  \hline  56 \quad 8 \quad 0 \quad 9 \text{ answer.} \\  \hline  \end{array}  $	$  \begin{array}{r}  \text{f. in.} \\  3 \quad 6 \\  - 2 \quad 3 \\  \hline  1 \quad 3 \text{ difference.} \\  \times 1 \quad 3 \\  \hline  1 \quad 3 \\  \hline  3) 1 \quad 6 \quad 9 \text{ square of the diff.} \\  \hline  6 \quad 3 \\  \hline  \end{array}  $
---	---

## PROBLEM IV.

To measure the frustum of a cone.

To the rectangle of the two diameters, add the sum of the squares of these diameters; multiply the product by .7854, and that product by the length; then one third of the last product will give the solidity.

*Note.* If the circumferences are given, proceed in the same manner, only multiply by .07958, instead of .7854.

## EXAMPLE.

*What is the solidity of the frustum of a cone, the diameter of the greater end being 3 feet, and that of the lesser end 2 feet, and the altitude 9 feet*

$$\begin{array}{rcl}
 3 \times 3 & = & 9 \\
 2 \times 2 & = & 4 \\
 2 \times 3 & = & 6 \\
 \hline
 & & 19 \\
 & & \hline
 & & 14.9226 \\
 & & 9 \\
 & & \hline
 & & 3)134.3034
 \end{array}
 \begin{array}{r}
 .7854 \\
 \times 19 \\
 \hline
 70686 \\
 7854 \\
 \hline
 14.9226 \\
 9 \\
 \hline
 3)134.3034
 \end{array}$$

44.7678 the solidity of the frustum.

OF  
A PRISMOID.

---

*DEFINITION.*

**A** PRISMOID is a solid contained under six planes, the ends being parallel, but unlike rectangles; and the other four sides, each opposite, two are equal trapeziums.

PROBLEM I.

To measure a prismoid ABCDEFG.

Multiply the length at the greater end BC, by the breadth at the lesser end FE, and the length at the lesser end DE, by the breadth at the greater end AB.

To half the sum of the two products, add the areas of the two ends; that sum multiplied by  $\frac{1}{3}$  of the height GH, gives the solidity.

EX-

## EXAMPLE.

*What is the solidity of a prismoid ABCDEFG, whose greater end is 12 inches by 8, and the lesser end 8 inches by 6, and the length or height HI, 5 feet?*

$$\begin{array}{r}
 12 \times 6 = 72 \\
 8 \times 8 = 64 \\
 \hline
 2)136 \\
 \hline
 68 \text{ half the sum of the products.} \\
 12 \times 8 = 96 \text{ the area of the greater end.} \\
 8 \times 6 = 48 \text{ the area of the lesser end.} \\
 \hline
 212 \text{ sum.} \\
 20 \text{ one third of the height.} \\
 \hline
 1728)4240(2 \text{ solid ft. and } 784 \text{ solid in. the ans.} \\
 3456 \\
 \hline
 784 \\
 \hline
 \end{array}$$

*Or by feet and inches.*

$$\begin{array}{r}
 1\text{f.} \times 6\text{i.} = 6\text{i.} \\
 8\text{i.} \times 8\text{i.} = 5\text{i. } 4\text{ii.} \\
 \hline
 2)11 \text{ } 4 \text{ sum.} \\
 \hline
 5\text{i. } 8\text{ii. half the sum of the products.} \\
 8\text{f.} \times 1\text{i.} = 8 \text{ the area of the greater end.} \\
 8\text{i.} \times 6\text{i.} = 4 \text{ 0 the area of the lesser end.} \\
 \hline
 1 \text{ } 5 \text{ } 8 \text{ sum.} \\
 \times 1 \text{ } 8 \text{ one third of the height.} \\
 \hline
 11 \text{ } 9 \text{ } 4 \\
 1 \text{ } 5 \text{ } 8 \\
 \hline
 2 \text{ } 5 \text{ } 5 \text{ } 4 \\
 \text{f. } \text{i. } \text{ii. } \text{iii.}
 \end{array}$$

PROBLEM

## PROBLEM II.

To find the solidity of a sphere or globe.

Multiply the cube of the diameter AB by 5236, and the product is the solidity.

### EXAMPLE.

*What is the solidity of a globe, whose diameter is 3 feet ?*

$$3 \times 3 \times 3 = 27$$

And  $\cdot 5236$   
 $27$

---

36652  
 10472

---

141372 the solidity of the globe.

## PROBLEM III.

To find the solidity of the segment of a globe.

To three times the square of half the diameter AB of the base, add the square of the height CD; multiply the sum by the height CD, then the product multiplied by  $\cdot 5236$ , will give the solidity.

EX-

## EXAMPLE.

*What is the solidity of a spherical segment, the diameter of the base being 4 feet, and the height of the segment 3 feet?*

$\frac{4}{2} = 2$  half the diameter.

$\begin{array}{r} 2 \\ \times 2 \\ \hline 4 \\ \times 3 \\ \hline 12 \\ + 9 \\ \hline 21 \end{array}$	$3 \times 3 = 9$ the square of the height.  $12$ three times the square of half the diameter. $+ 9$ <hr/> $21$ <hr/>
	$\begin{array}{r} 5236 \\ \times 21 \\ \hline 5236 \\ 10472 \\ \hline 109956 \end{array}$
	$10.9956$ the solidity. <hr/>

## PROBLEM IV.

To find the solidity of a spherical zone, the radius ED, and FB of the two parallel circles at the end being given, and their distance EF.

To the squares of the two radiuses, add one third of the square of the height; multiply the sum by the height, and the product by 1.5708, will give the solidity.

EX-

EXAMPLE.

*What is the solid content of a spherical zone, whose greater radius is 12 inches, and the lesser 10 inches; and the height or distance of the ends 4 inches?*

$$\frac{12^2 + 10^2 + \frac{4^2}{3}}{\times 4 \times 1.5708} = 1566.6112 \text{ the solidity required.}$$

PROBLEM V.

To find the solidity of a wedge ABCDE.

Multiply the area of the base ABC by the perpendicular height EF, and half the product will give the solidity.

EXAMPLE.

*Required the solidity of a wedge ABCDE, the side AB being 1f. 3in. and BC 2f. 6in. and the height 4f.?*

$$\text{Then } \frac{2.5 \times 1.25 \times 4}{2} = 6.25 \text{ the solidity.}$$

*Note.* The solidity of any prismatic ungula will be found in the same manner; that is, half the product of the area of the base multiplied into the height, will give the solidity.

## PROBLEM VI.

To find the solidity of the hoof, or ungula, from the frustum of a square pyramid.

To the square of the side of the base, or that end which is complete, add one half of the product of the sides of the two ends; this being multiplied by one third of the height, gives the solidity.

And if the hoofs are any other than that of a square pyramid, find the square root of the area of each end, which will give the side of a square equal in area; then proceed as above.



## EXAMPLE.

*Required the solidity of an ungula ABCDE, from the frustum of a square pyramid, the side of the greater end, which is complete, being 1f. 6in. that of the lesser end 1f. 3in. and the height FG 5f.?*

$$\text{Then } \overline{1.5}^2 + \frac{\overline{1.5} \times \overline{1.25}}{2} \times \frac{5}{3} = 5.3125 \text{ the solidity.}$$

## PROBLEM VII.

To find the solidity of a spheroid; the fixt axis and the revolving axis being given.

Multiply the fixt axis by the square of the revolving axis, and the product by .5236, will give the solidity.

EX-

EXAMPLE I.

*What is the solidity of a prolate spheroid, whose transverse axis is 100 feet, and the conjugate 60 feet?*

$$100 \times 60^2 \times .5236 = 188496 \text{ the solidity required.}$$

EXAMPLE II.

*What is the solidity of an oblate spheroid, whose transverse axis is 100 feet, and the shortest axis 60 feet?*

$$60 \times 100^2 \times .5236 = 314160 \text{ the solidity required.}$$

PROBLEM VIII.

To find the solidity of a parabolic conoid; the diameter AB of the base being given, and the perpendicular height CD.

Multiply the square of the diameter of the base by .3927, and the product by the height will give the solidity.

EXAMPLE.

*What is the solidity of a parabolic conoid, whose height is 50 feet, and the diameter of the base 30 feet?*

$$30^2 \times .3927 \times 50 = 17671.5 \text{ the solidity required.}$$

### PROBLEM IX.

To find the solidity of the frustum of a parabolic conoid, the greater diameter AB, the lesser CD, and the perpendicular height EF being given.

To the square of the diameter of the greater end AB, add the square of the diameter of the lesser end CD; multiply the sum by .3927, and the product by the height EF, will give the solidity required.

### EXAMPLE.

*What is the solidity of a parabolic frustum, the diameter of the greater end being 60 feet, the lesser end 48 feet, and the distance of the ends 18 feet?*

$60^2 + 48^2 = 5904$  the sum of the squares of the ends.

Then  $5904 \times .3927 \times 18 = 41733.0144$  the solidity required.

OF AN  
ANNULUS,  
OR  
CYLINDRIC RING.

DEFINITIONS.

I. **I**F a circle is carried round a right line as an axis, and in the same plane with the circle, either touching the axis, or at a given distance from it, it will generate a solid, called an annulus, or cylindric ring.

II. The diameter of the generating circle is called the thickness of the ring.

III. Twice the distance of the generating circle, from the axis of rotation, is called the inner diameter.

PROBLEM I.

To find the solidity of an annulus, or ring, whose thickness and inner diameter are known.

METHOD I.

To the thickness of the annulus, add the inner diameter; multiply the sum by the square of the thickness, and the product by 2.4674, will give the solidity sought.

METHOD

## METHOD II.

Multiply the circumference round the middle of the annulus, or that circle generated by the centre of the generating circle, by the area of the generating circle, and the product will give the solidity.

*Note.* This last method will give the solidity of any part of an annulus, or ring, comprehended between any two planes passing through the fixed axis.

## EXAMPLE.

*What is the solidity of an annulus, whose inner diameter is 8 inches, and the thickness of the annulus 3 inches?*

Then  $\overline{8 + 3} \times 3^2 \times 2.4674 = 244.2726$  the solidity.

## PROBLEM II.

To find the solidity of a hollow cylinder, the exterior and interior diameters being given, and the perpendicular height.

Multiply the sum of the diameters CD and EF by their difference, and the product by .7854; then by the altitude GH of the cylinder, and you will have the solidity required.

EX-

EXAMPLE.

*Required the solidity of a hollow cylinder, the exterior diameter CD being 14 feet, the interior diameter EF 12 feet, and the perpendicular height AC 10 feet.*

Then  $\overline{14 + 12} \times \overline{14 - 12} \times .7854 \times 10 = 408.408$   
the solidity.

PROBLEM III.

To find the solidity of a frustum of a hollow cone, of an equal thickness; the exterior and interior diameters at each end being given, and the perpendicular altitude.

Multiply together the sum of the two interior diameters, and the difference of the diameters at either end, by .7854, and by the perpendicular altitude; then by the said difference, and the continued product will give the solidity required.

EXAMPLE.

*Required the solidity of a frustum of a hollow cone, the bottom diameters, AB and GH, being respectively 35 and 32 feet; the top diameters, CD and EF, respectively 29 and 26 feet; and the perpendicular altitude IK, 25 feet.*

Then  $35 - 32 = 3$  the difference of the diameter.

Or  $29 - 26 = 3$  the difference of the diameter.

And  $\overline{32 + 26} + 3 \times .7854 \times 3 \times 25 = 3593.205$   
the solidity.

PROBLEM

## PROBLEM IV.

To find the solidity of a hollow segment of a globe, the bottom diameters AB and CD being given, and the perpendicular altitudes DF and EG of the exterior and interior segments.

Find the solidity of each segment by Prob. III. p. 199, subtract the lesser segment from the greater, and the difference will give the solidity of the hollow segment.

## PROBLEM V.

To find the solidity of a hollow frustum of a zone; given the bottom diameters AB and CD, the top diameters EF and GH, and the perpendicular altitude IK.

Add into one sum, twice a rectangle under the difference of the diameters at the bottom, by the less diameter; twice a rectangle of the difference of the top diameters by the less, and the squares of the difference of both ends; then multiply the sum by .7854, and the product by the perpendicular altitude will give the solidity.

## EXAMPLE.

*Required the solidity of a hollow zone, the diameters AB and CD, at the bottom, being respectively 26 and 24 feet; and the top diameters, EF and GH, 23 and 20 feet respectively; and the perpendicular height IK, 4 feet.*

$$2 \times 26 - 24 \times 24 + 2 \times 23 - 20 \times 20 + 26 - 24^2 + 23 - 20^2 \\ \times .7854 \times 4 = 719.4264 \text{ the solidity.}$$

OF

OF FINDING THE  
CONVEX SURFACES OF SOLIDS.

---

PROBLEM I.

To find the convex surface of a right cylinder, the circumference and length of the cylinder being given.

· Multiply the circumference by the length of the cylinder, and the product will be the area.

EXAMPLE.

*What is the convex superficies of a right cylinder, whose circumference is 2ft. 6in. and the length 5ft. 3in.?*

Then  $\overline{5\text{f. } 3\text{i.}} \times \overline{2\text{f. } 6\text{i.}} = 13\text{f. } 1\text{i. } 6\text{ii.}$  the answer.

*Note.* If the diameter is given, find the circumference, and proceed as before.

## PROBLEM II.

To find the convex superficies of a right cone, the circumference and slant side being given.

Multiply the circumference by the slant side of the cone, and half the product will be the area.

### EXAMPLE.

*What is the convex superficies of a right cone, the circumference of the base being 4.5 ft. and the slant side 6.25 ft.?*

Then  $\frac{6.25 \times 4.5}{2} = 14.0625$  the superficies required.

*Note.* If the diameter is given, find the circumference, and proceed as before.

## PROBLEM III.

To find the convex surface of the frustum of a right cone, the circumferences of both ends being given, and the slant side of the cone.

Multiply the sum of the circumferences by the slant side of the cone, and half the product will be the area.

EX-

EXAMPLE.

*What is the convex superficies of a frustum of a right cone, the circumference of the base being 4·6 ft. the circumference of the top being 3·25 ft. and the altitude 5·75 feet?*

$$\text{Then } \frac{4\cdot6 + 3\cdot25}{2} \times 5\cdot75 = 22\cdot56875 \text{ the answer.}$$

PROBLEM IV.

To find the superficies of a sphere or globe, the greatest circumference being given.

Multiply the square of the circumference by ·3183, and the product will be the superficies.

EXAMPLE.

*What is the superficies of a globe, the greatest circumference being 10·6 ft.?*

$$\text{Then } 10\cdot6 \times 10\cdot6 \times \cdot3183 = 35\cdot764188 \text{ the superficies required.}$$

PROBLEM V.

To find the convex superficies of the segment of a sphere or globe, the diameter of the base of the segment, and its height, being given.

To the square of the diameter of the base, add the square of twice the height; and the sum multiplied by ·7854, will give the superficies.

## EXAMPLE.

*What is the convex surface of the segment of a globe, the diameter of the base being 17.25 ft. and the height 4.5 ft.?*

$$2 \times 4.5 = 9 \text{ twice the height.}$$

$$9 \times 9 = 81 \text{ square of twice the height.}$$

$$\overline{17.25}^2 = 297.5625 \text{ square of the diameter of the base.}$$

Then  $297.5625 + 81 \times .7854 = 297.3229875$  the superficies required.

## PROBLEM VI.

**To find the convex surfaces of a spherical zone, the diameters of the ends and their distance being given.**

Find the diameter of the sphere by Prob. XIII. and Prob. XIV. in Mensuration of Superficies; then multiply the diameter of the sphere, and the distance of the parallel ends of the zone together, and the product by 3.1416, will give the superficies required.

## EXAMPLE.

*In a spherical zone, the distance of the parallel ends being 4 in. the diameter of the greater end 24 in. and that of the lesser end 20 in. what is the convex superficies, when the centre of the sphere is without the zone?*

The distance of the greater chord from the centre, will be found to be 3.5 inches, by Prob. XIII.

The radius will be found to be 25 inches, by Prob. XIV. or the diameter 50 inches.

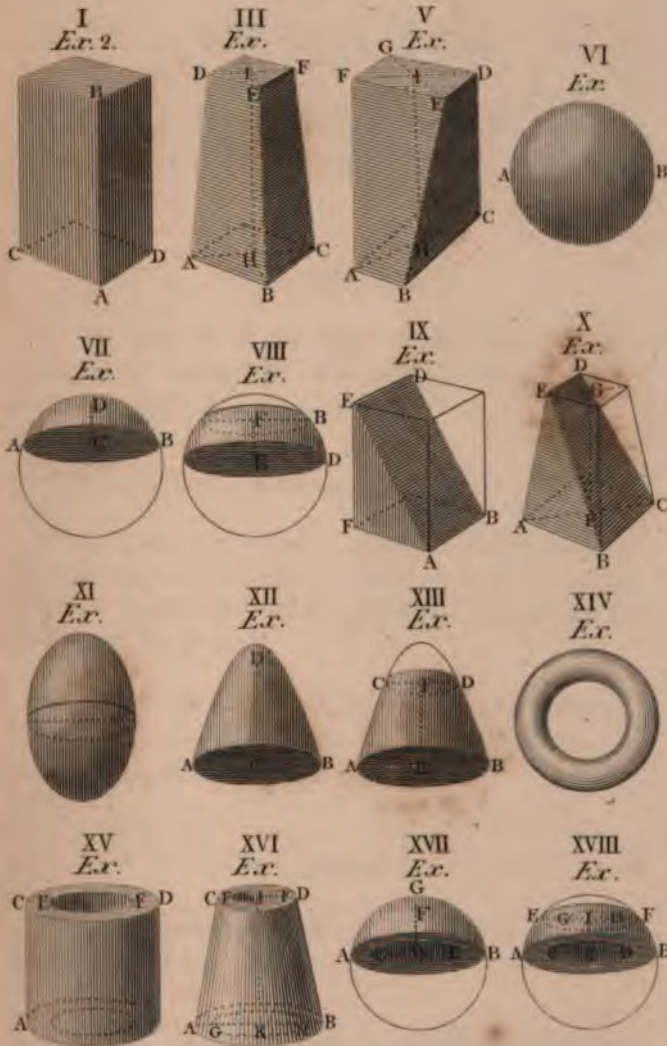
$$\text{Then } 50 \times 4 \times 3.1416 = 628.32 \text{ the answer.}$$

*Note.* If the diameter is given, find the circumference, and proceed as before.

## PROBLEM

# MENSURATION of SOLIDS

## PROBLEMS





## PROBLEM VII.

To find the convex superficies of an annulus, or ring, whose thickness and inner diameters are known.

## METHOD I.

To the thickness of the ring, add the inner diameter; multiply the sum by the thickness, and the product by 9.869, will give the superficies required.

## METHOD II.

Multiply the circumference of the generating circle by the circumference round the middle of the ring, or that line generated by the centre of the generating circle, and the product will be the area.

*Note.* This last method will give the convex superficies of any part of an annulus, or ring, comprehended between two planes passing through the fixt axis.

## EXAMPLE.

*What is the convex superficies of an annulus, or ring, whose inner diameter is 8in. and the thickness 3in.?*

Then  $\overline{3 + 8} \times 3 \times 9.869 = 325.677$  the superficies required.

OF

## SPECIFIC GRAVITY.

## DEFINITION.

I. The specific gravity of a body, is the relation that the weight of a magnitude of that kind of body, has to the weight of an equal magnitude of another kind of body.

II. In this comparison of the weights of bodies, it is convenient to consider one body as the standard or unit, to which the others are to be compared; and as rain water is nearly alike in all places, therefore this seems to be the most convenient for a standard.

III. It has been found by repeated experiments, that a cubic foot of rain water weighed  $62\frac{1}{2}$  pounds avoirdupoise; consequently,  $\left(\frac{62\cdot5}{1728}\right)$  0·03616898lb. is the weight of one cubic inch of rain water.

IV. The knowledge of the specific gravities of bodies, is of great use in computing the weights of such bodies, as are too heavy or too difficult to have their weight discovered by other means.

A TABLE,

# A TABLE,

Shewing the specific gravity to rain water, of metals, and other bodies; and the weight of a cubic inch of each, in parts of a pound avoirdupoise, and an ounce troy.

Bodies.	Sp. gra.	Wt. lb. av.	Wt. oz. tr.
Fine gold - - -	19·640	0·7103587	10·359273
Standard gold - -	19·520	0·7060185	9·962625
Coast gold - - -	18·888	0·6828703	9·911707
Quicksilver - - -	13·762	0·4976574	7·384411
Lead - - - - -	11·313	0·4091696	5·984010
Fine silver - - -	11·092	0·4011501	5·850035
Standard silver -	10·629	0·3844400	5·556769
Cast silver - - -	10·528	0·3807870	5·503967
Copper - - - -	8·769	0·3171658	4·747121
Plate brass - - -	8·350	0·2942593	4·404273
Cast brass - - -	8·104	0·2929832	4·272409
Steel - - - - -	7·850	0·2839265	4·142127
Bar iron - - - -	7·764	0·2808159	4·031361
Block tin - - - -	7·238	0·2617901	3·861519
Cast iron - - - -	7·135	0·2580647	3·806568
Loadstone - - -	5·106	0·1846788	2·724083
Blue slate - - -	3·500	0·1264914	1·867272
Veined marble - -	2·702	0·0977286	1·429411
Common glass - -	2·600	0·0940393	1·360841
Flint stone - - -	2·582	0·0933883	1·351419
Portland stone - -	2·570	0·0929543	1·345139
Free stone - - -	2·352	0·0915788	1·231038
Brick - - - - -	2·000	0·0723379	1·046801
Alabaster - - - -	1·888	0·0683061	0·988456
Ivory } - - - -	1·832	0·0662606	0·958489
Horn } - - - -			
Brimstone - - -	1·800	0·0651042	0·949424
Clay - - - - -	1·712	0·0619213	0·902498

TABLE

TABLE CONTINUED.

Bodies.	Sp. gra.	Wt. lb. av.	Wt. oz. tr.
Lignum vitæ - -	1·327	0·0479862	0·699936
Coal - - - -	1·255	0·0453921	0·661959
Pitch - - - -	1·150	0·0415943	0·606576
Mahogany wood -	1·063	0·0384475	0·560691
Dry box wood - -	1·030	0·0372530	0·543272
Milk } - -	1·030	0·0372530	0·543272
Sea water } - -	1·000	0·0361690	0·527458
Rain water - - -	0·993	0·0359158	0·523766
Red wine - - -	0·995	0·0359881	0·524820
Bees wax - - -	0·932	0·0337095	0·491591
Linseed oil - - -	0·927	0·0335503	0·489268
Proof spirits or } brandy - - }	0·915	0·0330946	0·489008
Dry oak - - -	0·913	0·0330222	0·481569
Olive oil - - -	0·854	0·0308883	0·450449
Beech - - - -	0·800	0·0289352	0·421966
Dry elm } - - -	0·800	0·0289352	0·421966
Dry ash } - - -	0·800	0·0289352	0·421966
Dry wainscot - -	0·747	0·0270182	0·394011
Dry yellow fir - -	0·657	0·0237630	0·346539
Cedar - - - -	0·613	0·0221715	0·323332
Dry white deal -	0·569	0·0205801	0·300123
Cork - - - -	0·240	0·0186805	0·126590
Air - - - -	0·0012	0·0000434	0·000633

*Note.* 7000 grains make 1lb. avoirdupoise, and 5760 grains make 1lb. troy; therefore as 1lb. avoirdupoise  $\div$  1lb. troy  $\therefore$  7000  $\div$  5760, or as 700  $\div$  576, consequently, 1lb. avoirdupoise, multiplied by  $\frac{576}{700}$  gives 1lb. troy, and 1lb. troy, multiplied by  $\frac{700}{576}$  gives 1lb. avoirdupoise.

For

For example, 12 ounces is a pound troy; then  $\frac{700}{576} \times 12 = 14.58\frac{1}{3}$  the number of troy ounces in one pound avoirdupoise, and  $14.58\frac{1}{3}$  multiplied into any number under Wt. lb. av. in the table, will give its opposite number under Wt. oz. tr.; on the contrary, if  $\frac{567}{12 \times 700}$  be multiplied by any number under Wt. oz. tr., it will produce its opposite or horizontal number under Wt. lb. av.

## PROBLEM I.

The weight of a body being given, to find its solidity.

Divide the given weight in pounds avoirdupoise, by the tabular weight corresponding to the name of the same kind, and the quotient will be the solidity in cubic inches; and if the quotient is divided by 1728, you will have the number of cubic feet.

### EXAMPLE.

*What is the solidity of a block of marble, weighing 8 tons 14cwt. in cubic feet?*

Now 8 tons 14 cwt  $\approx$  19488lb.

Then  $\frac{19488}{.0977286} \div 1728 = 115.4$  cubic feet the solidity.

716

PROBLEM.

PROBLEM II.

The linear dimensions, or solidity, of a body being given, to find its weight.

Multiply the cubic inches contained in the body, by the tabular weight corresponding to the mark of the tuncs used, and the product will give the weight in pounds avoirdupois.

EXAMPLE.

What is the weight of a piece of iron, of a rectangular form, whose length is 50in. the breadth 18in. and the depth 12in.?

Now  $56 \times 18 \times 12 = 12096$  inches.  
Then  $12096 \times .0330946 = 4003122161b$ , the weight.

OF THE

## FIVE REGULAR SOLIDS.

### DEFINITIONS.

I. A regular solid, is a body that either may be inscribed or circumscribed by a sphere, in such a manner as to be contained under equal and similar planes; alike posited, and equally distant from the centre of the sphere.

II. The

II. The *Tetraedron*, is contained under four equilateral triangles.

III. The *Hexaedron*, is contained under six equal squares.

IV. The *Octaedron*, is contained under eight equilateral triangles.

V. The *Dodecaedron*, is contained under twelve equilateral and equiangular pentagons.

VI. The *Icosaedron*, is contained under twenty equilateral triangles.

## PROBLEM I.

To find the superficies, and solidity, of any of the five regular bodies.

*To find the superficies.*

Multiply the area (taken from the following table) by the square of the linear edge of the solid, for the superficies.

*To find the solidity.*

Multiply the tabular solidity by the cube of the linear edge, for the solid content.

Surfaces and Solidities of the five regular Solids.			
No. of sides.	Names.	Surfaces.	Solidities.
4	Tetraedron - -	1.73205	0.11785
6	Hexaedron - -	6.00000	1.00000
8	Octaedron - - -	3.46410	0.47140
12	Dodecaedron - -	20.64573	7.66312
20	Icosaedron - -	8.66025	2.18169

## EXAMPLE I.

*If the linear edge or side of a tetraedron be 3, required its superficial and solid content.*

Thus  $1.73205 \times 9 = 15.58845$  superficies.

And  $0.11785 \times 27 = 3.18195$  solidity.

## EXAMPLE II.

*What is the surface and solidity of the hexaedron, whose linear side is 2?*

Answer  $\left\{ \begin{array}{l} \text{superficies} = 24 \\ \text{solidity} = 8 \end{array} \right\}$

## EXAMPLE III.

*Required the superficies and solidity of the octaedron, whose linear side is 2.*

Answer  $\left\{ \begin{array}{l} \text{superficies} = 13.8564 \\ \text{solidity} = 3.7712 \end{array} \right\}$

## EXAMPLE IV.

*What is the superficies and solidity of the dodecaedron, whose linear side is 2?*

Answer  $\left\{ \begin{array}{l} \text{superficies} = 82.58292 \\ \text{solidity} = 61.30496 \end{array} \right\}$

## EXAMPLE V.

*What is the superficies and solidity of an icosaedron, whose linear side is 2?*

Answer  $\left\{ \begin{array}{l} \text{superficies} = 34.641 \\ \text{solidity} = 17.45352 \end{array} \right\}$

OF

MEASURING

## IRREGULAR SURFACES & SOLIDS.

### DEFINITION.

An irregular surface or solid, is such a surface or solid which have their bounds by lines or surfaces in any manner whatever, of no particular kind of form or shape, but merely accidental, according as they are to be found or given.

### PROBLEM I.

To measure any irregular surface whatever, by means of equidistant ordinates.

#### METHOD I.

To the half sum of the two outside ordinates, add the sum of all the other remaining ordinates; multiply the whole sum by the distance between any two ordinates, and the product will be the superficial content.

EX-

## EXAMPLE I.

Let Fig. 1, Plate 56, be the curve proposed, whose equidistant ordinates, AB, CD, EF, GH, IK, LM, and NO, are respectively 5ft. 5ft. 6in. 6ft. 7ft. 8ft. and 8ft. and the distance of AC, CE, EG, or GI, is 3ft. required the area of the curve.

$$\begin{array}{r} AB = 5 \\ NO = 8 \end{array}$$

2)13

$$\begin{array}{r} 6 \quad 6 \text{ half the sum of the outside ordinates.} \\ CD = 5 \quad 6 \\ EF = 6 \quad 0 \\ GH = 7 \quad 0 \\ IK = 9 \quad 0 \\ LM = 10 \quad 0 \end{array}$$

$$\begin{array}{r} 64 \quad 0 \\ 3 \end{array}$$

132 superficies.

## EXAMPLE II.

Plate 54, Fig. 2, let ABCD be a circle, whose diameter AC, or BD, is 10ft. it is required to find the area by means of equidistant ordinates, marked 3ft. 4ft. 4.5ft. 4.9ft. and 5ft. being at the distance of 1 foot from each other.

$$\begin{array}{r} 0 \\ 5 \end{array}$$

$$2)5$$

2.5 half sum of the outside ordinates.

$$\begin{array}{r} 3 \\ 4 \\ 4.5 \\ 4.9 \end{array}$$

18.9 area of one quarter.

$$4$$

75.6 feet, area of the whole.

If

If the diameter, which is 10 feet, be multiplied by  $\cdot 7854$ , the product,  $78\cdot 54$ , will be the area. From hence it appears, that this mode of operation, by means of equidistant ordinates, is exceedingly near the truth in measuring irregular planes; for it will produce the area of a circle, which is one of the most oblique curves possible, as the ends raise quite perpendicular to the axis, from only 10 equidistant spaces within the  $\frac{1}{16}$  part of the truth; and would be still nearer when applied to measuring any plane surface, where it is bounded partly by concave and partly by convex curves: because, if wholly bounded by a convex curve, or curves, the area will be something less than the truth; but if bounded by a concave curve, or curves, the area will be something greater than the truth; and if the extremities of the ordinates are joined by straight lines, the area so found will be exactly true: but the following is a method of approximation still nearer to the truth, whether the curve be concave or convex to the axis.

#### METHOD II.

Divide the given curve, by ordinates, into any even number of equal parts; then add into one sum four times the sum of all the even ordinates; twice the sum of all the odd ordinates, except the first and last, and also the first and last ordinates; and if one third of that sum is multiplied by the common distance between any two ordinates, the product will be the answer.

EX-

## EXAMPLE I.

Let Fig. 1, Plate 54, be a curve of any kind, whose equidistant ordinates, AB, CD, EF, GH, IK, LM, and NO, are respectively 5ft. 5ft. 6in. 6ft. 7ft. 9ft. 10ft. and 8ft. and the distance between the ordinates is 3ft. required the area of the curve.

CD, GH, and LM, will be the even ordinates; that is, the second, fourth, and sixth; EF, and IK, the odd ordinates, that is, the third and fifth; AB, and NO, the first and last.

	f.	in.		f.
CD =	5	6	EF =	6
GH =	7	0	IK =	9
LM =	10	0		—
	22	6		15
		4		2
		—		—
				30
90	4	four times the sum of the even ordinates.		
30	0	twice the sum of the odd ordinates.		
5	0	first ordinate.		
8	0	last ordinate.		
		—		
3)133	0	sum.		
	44	4		
		3		
		—		
133	0	the area, or superficial content.		

Now by comparing this area, viz. 133 feet, with the area found in Method I. Example I. viz. 132 feet, there appears to be a difference of 1 foot; but this last method is the most correct.

EX-

EXAMPLE II.

Plate 54, Fig. 3, let ACEGILN be a concave curve, whose equidistant ordinates, A, BC, DE, FG, HI, KL, and MN, are respectively 0, 1, 3, 6, 10, 15, 21, and the common distance 2; required the area.

BY METHOD I.

$$\begin{array}{r}
 0 \\
 21 \\
 \hline
 2)21 \\
 \hline
 10 \cdot 5 \\
 1 \\
 3 \\
 6 \\
 10 \\
 15 \\
 \hline
 45 \cdot 5 \\
 2 \\
 \hline
 \end{array}$$

91·0 the area, greater than the truth.

BY METHOD II.

$$\begin{array}{r}
 3 \\
 10 \\
 \hline
 13 \\
 2 \\
 \hline
 26
 \end{array}
 \qquad
 \begin{array}{r}
 1 \\
 6 \\
 15 \\
 \hline
 22 \\
 4 \\
 \hline
 \end{array}$$

88 four times the sum of the even ordinates.

26 twice the sum of the odd ordinates.

0 first ordinate.

21 last ordinate.

$$\begin{array}{r}
 3)135 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 45 \\
 \hline
 \end{array}$$

2 common distance.

90 the area, very near the truth.

## EXAMPLE VIII.

Let  $AHI$  be a parabola, whose ordinates,  $A, BC, DE, FG,$  and  $HI,$  are respectively 0, 7, 12, 15, and 16, and their common distance 5; required the area of the curve.

7	12
15	2
—	—
22	24
4	
—	

88 four times the sum of the even ordinates.

24 twice the sum of the odd ordinates.

16 sum of the end.

3)128

42 $\frac{2}{3}$   
6

256 the true area.

There is another method for finding the areas of curvilinear spaces, besides what has already been shown, which is as follows: divide the sum of all the ordinates by the number of them, for a mean breadth, which is to be multiplied by the length for the content; but this rule is a very false one; it gives the area by far too great when the curve is concave, and by far too small when it is convex, and will not give the true area in any case whatever, except the curve become a straight line; in which case, all the other rules will coincide with it. But in order to show the falsity of this rule, suppose it were required to find the contents of the same figure, as in the last Example, then the sum of all the ordinates, viz.  $0+7+12+15+16=50$ , their number is 5; and 50 divided by

by 5, is equal to 10: this being multiplied by the whole length, viz. 24, gives 240 instead of 256, the exact area of the curve found by the last Example; the difference of this Example being too small by  $\frac{1}{16}$  part of the true area, which is very considerable.

## PROBLEM II.

To find the superficial content of a mixed figure, partly a curve, and partly right lined.

Find the area of the curve part of the figure by the last Problem, by dividing it into equidistant ordinates; divide the right-lined parts of the figure by ordinates drawn through every angle, which will divide the right-lined part of the figure into trapezoids and triangles; find the area of each part, according to their respective rules; add the areas of all the parts together, and the sum will give the area of the whole figure.

### EXAMPLE I.

*Plate 54, Fig. 5, let ABKNORS be the figure proposed, to find its area.*

As the end AB turns round very perpendicular to the base AS, draw the ordinate CB in such a manner, as it may cut off the most perpendicular part of the curve AB at the end, and divide it by ordinates, which are respectively 1, 2,  $1\frac{1}{2}$ , 1, 0, at the distance of 3 from each other; the part CBKL of the curvilinear space is also divided into four equal parts, between the first and last ordinates BC, KL, by the ordinates ED, GF, HI, and LK, which are respectively 12, 13, 12, 10, and 9, and their common distance 4; the other

g g 2

parts

parts of the figure are divided into three trapezoids, KLMN, MNOP, OPQR, and the triangle QRS, by ordinates from the angles at K, N, O, and R; the whole figure being thus prepared, by dividing it into curvilinear spaces, trapezoids and a triangle, each part will be measured according to their respective rules. The measures or dimensions are marked on their respective places on the figure; the contents of each part is computed separately, as is shown in the following operation.

$\begin{array}{r} 1\frac{1}{2} \\ 2 \\ \hline 3 \end{array}$	$\begin{array}{r} 12 \\ 2 \\ \hline 24 \end{array}$	
$\begin{array}{r} 2 \\ 1 \\ \hline 3 \\ 4 \\ \hline 12 \\ 3 \\ 1 \\ \hline 3)16 \\ \hline 5\frac{1}{3} \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ 10 \\ \hline 23 \\ 4 \\ \hline 92 \\ 24 \\ 12 \\ 9 \\ \hline 3)137 \\ \hline 45\cdot6 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 16 \\ \hline 2)25 \\ \hline 12\frac{1}{2} \\ 4 \\ \hline \end{array}$
16 area of the part ABC.	182·6 area of the part CBKL.	
$\begin{array}{r} 16 \\ 13 \\ \hline 2)29 \\ \hline 14\cdot5 \\ 3 \\ \hline 27\cdot0 \\ 49\cdot0 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ 14 \\ \hline 2)27 \\ \hline 13\frac{1}{2} \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 2)14 \\ \hline 7 \\ 7 \\ \hline \end{array}$
43·5	43·5 area of MNOP.	49 area of the triangle QRS.
368·1	sum of the areas, or contents of the whole figure.	

PROBLEM

### PROBLEM III.

To measure any irregular figure, bounded wholly by right lines.

Divide the whole figure into trapeziums and triangles, divide each trapezium into two triangles, by means of diagonals from the other angles, let fall perpendiculars to the diagonals; then to find the area of any of the trapeziums, multiply the half sum of the two perpendiculars in that trapezium by its diagonal; find the area or content of all the trapeziums in this manner, and of the triangles, if any; add their several areas together, and the sum will give the area of the whole figure.

#### EXAMPLE.

*Plate 54, Fig. 6, let ABCDEFG be the figure proposed, which is divided into two trapeziums ABFG, BCEF, and a triangle CDE; the diagonal BG of the trapezium ABFG is 16, and the two perpendiculars AH and IF are respectively 4 and 8; the diagonal BE of the trapezium BCEF is 10, and the perpendiculars KF and CL are respectively 14 and 6; the base CE of the triangle CDE is 8, and its perpendicular 5, required the area of the whole figure ABCDEFG.*

4	14	5
8	6	8
<hr/>	<hr/>	<hr/>
2)12	2)20	2)40
<hr/>	<hr/>	<hr/>
6	10	20
16	10	
<hr/>	<hr/>	
96	100	

Then 96, the area of the trapezium ABFG.

100, the area of the trapezium BCDF.

20, the area of the triangle FDE.

And 216, the area of the whole figure ABCDEFG.

PROBLEM

## PROBLEM IV.

To find the solidity of a solid, by means of equidistant sections or planes.

Divide the length of the solid into any even number of equal parts; find the area of all the parallel sections passing through these parts perpendicular to the axis of the solid; then to four times the sum of the areas of all the even planes, add twice the sum of the areas of all the odd planes, except the first and last, and the areas of the two ends; divide the sum by 3, multiply the quotient by the common distance, and the product will give the solidity.

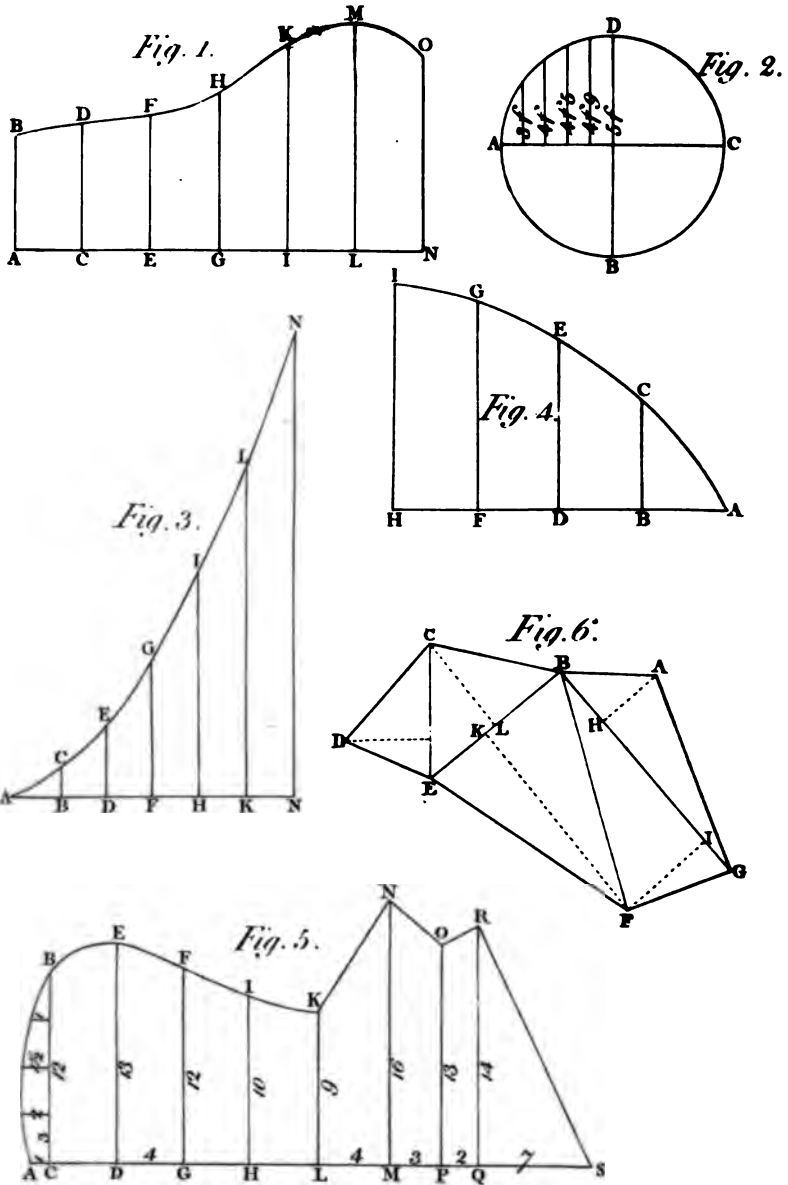
N. B. If the sections are circular, the rule may be as follows: to four times the area of the even planes, add twice the sum of the areas of the odd planes, excepting the first and last, and the sum of the areas of the ends, or the first and last planes; then multiply the sum by  $\cdot 2618 = \frac{7854}{3000}$ , and that product by the common distance, and it will give the solidity sought.

## SCHOLIUM.

This Problem is accurately true for parabolic curves, or solids, generated from the revolution of conic sections, or right lines. For all kinds of pyramids, or frustums of pyramids, or any other kind of areas and solidities, it is a very near approximation.

It is evident, that the greater the number of ordinates or sections are used, the more accurate will the area or solidity

# MENSURATION.



1

2

3

4

5

6

solidity be determined: but in practice a few sections will be found sufficient to answer the purpose. This is the best method that possibly can be devised for the practice of gauging, or for measuring curved timber trees, or the like unequally thick; or any kind of curved solids whatever, generated about an axis; for when all other methods fail, this is the only one that can be depended upon for its accuracy.

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OF

## MEASURING TIMBER.

---

### PROBLEM I.

To measure timber scantling.

Find the area at either end, and multiply it by the length, will give the solidity.

#### EXAMPLE I.

*Suppose a joist is 4in. by 9in. and 8ft. long, what is the solidity?*

in.	
4	
9	
<hr/>	
36	00
8	
<hr/>	
2	0 feet, the solidity.

EX-

## EXAMPLE II.

*What is the solidity of a joist 3½ in. by 9 in. and 10 f. long?*

i.	ii.
3	6
	9i.
<hr/>	
2i.	7ii. 6iif.
	10
<hr/>	
feet 2	2 3 0iii.

## PROBLEM II.

To measure timber trees, or unsquared timber,  
equally thick.

## METHOD I.

Multiply the square of  $\frac{1}{4}$  of the trees compass for the side of a square equal in area to the end of the tree, by the length of the tree, and the product will give the solidity.

This method, though easy in practice, is very erroneous in principle, as the content by this rule is too small by above one fourth of itself.

The true rules for measuring round timber, have been already given for measuring a cylinder: but if this rule should be thought troublesome, the following is a method which will come very near the truth, and nearly as expeditious in practice as the above method, and therefore may be esteemed true.

## METHOD II.

Multiply the square of  $\frac{1}{4}$  of the trees compass by the length of the tree, and double the product will be the content.

MEN-

MENSURATION  
OF  
*ARTIFICERS' WORKS,*  
CONCERNED IN  
**BUILDING.**

---

*The Artificers' Works which are treated on here, are Bricklayers, Carpenters, Joiners, Masons, Glaziers, Painters, Paviers, Plasterers, Plumbers, and Slaters.*

Artificers compute the quantity, or contents of their work, by several different measures, as Glazing and Masonry, by the foot; Painting, Plastering, Paving, &c. by the yard, of 3 feet square, or 9 square feet.

Bricklayers compute the quantity of their work by a rod of  $16\frac{1}{2}$  feet square, or  $272\frac{1}{4}$  square feet, at one brick and a half thick, which Bricklayers call the standard thickness; a rod of  $5\frac{1}{2}$  yards square, that is,  $30\frac{1}{4}$  square yards, at  $1\frac{1}{2}$  brick thick; but although  $272\frac{1}{4}$  is a rod of brick work, yet the  $\frac{1}{4}$  is always omitted by measurers, and therefore 272 square feet is commonly called a rod of brick work.

All works, whether superficial or solid, are computed by the rules proper for the figure of them.

The most common instruments for taking the measures are, a five feet rod, divided into feet, and quarters of a foot; and a rule, either divided into inches, or 12 parts, and each 12th part into 12 others: a fractional part beyond this division, Measurers seldom, or never, take any account of.

When the dimensions are taken, by a rule divided in this manner, the best methods to square the dimensions will then be by duodecimals, by the rule of practice, or by the multiplication of vulgar fractions: but, in my opinion, the best method of taking dimensions is with a rule, when each foot is divided into ten parts, and each part into ten other parts, or seconds, because the dimensions may be then squared by the rules of multiplication of decimals, which is by far the shortest and readiest method. Those who contend that duodecimals, or cross multiplication, is the easiest method of squaring dimensions, as well as the most exact, are very much mistaken in their assertion; for if the dimensions are taken in duodecimals, and reduced to decimals, and then squared, the operation, in this case, will be much longer than if it had been done by decimals, and sometimes not so exact: but if the dimensions are taken in feet, 10ths, &c. the operation will not only be easier and shorter, but in many cases will be much more exact than duodecimals. The reason is obvious to those who consider, that there are many cases in which it will be impossible to express, truly, a decimal scale equal to a duodecimal one; neither will it, in many cases, be possible to express accurately a duodecimal scale equal to a decimal one; duodecimals have the same property with regard to 12th parts, as decimals have to 10th parts; therefore, in many cases, duodecimals will sometimes circulate and run on, *ad infinitum*, when reduced from decimals, as decimals will, when reduced from duodecimals: and farther, since duodecimals are expressed by a  
series

series of 12th parts, and decimals by a series of 10th parts, in multiplying each of the parts of the former, the trouble of dividing by 12 will then be unavoidable, and more burdensome to the mind than if the operation had been done by the latter, where there is no such division to be made, but merely to multiply, as in common multiplication, and point off the decimal places in the product.

This last method I shall always prefer, being the most natural, as well as the most easy of the two.



OF

## BRICKLAYERS' WORK.



### PROBLEM I.

To find the number of rods contained in a piece of brick work.

#### METHOD I.

If the wall is at the standard thickness, that is, one brick and a half thick, divide the area, or superficial content, by 272, and the quotient, if any, will be the answer in rods; and the remainder, if any, in feet: but if the wall is more or less than one brick and a half in thickness, multiply the area of the wall by the number of half bricks contained in its thickness; divide the product by 3, and the wall will be reduced to the standard thickness of a brick and a half in thickness; then divide the quotient by 272, and the last quotient will be the number of rods of brick work required.

H h 2

METHOD



## METHOD II.

Multiply the length of the wall by its breadth, and the product by its thickness; that is, to find the solidity of the wall: then multiply the solidity by 8, and divide the product by 9, and the quotient will be reduced to the standard thickness of one brick and a half in thickness; then divide by 272, as before, and the answer will be the number of rods contained in the brick work.

## EXAMPLE I.

*How many rods are there in a wall 62½ ft. long, 14 ft. 8 in. high, and 2½ bricks thick?*

f.	i.	
62	6	
14	8i	
<hr/>		
41	8	
875	0	
<hr/>		
916	8	
		5 number of half bricks in thickness.
<hr/>		
3)4583	4	
<hr/>		
272)1527	9	4(5 167 9 4
1360		
<hr/>		
167		

In a piece of brick work, of the aforesaid dimensions, there are 5 rods, 167 feet, 9 inches, and 4 seconds; but in finding the number of rods in a piece of brick work, neither inches nor seconds need be taken any account of, as such an addition is so very trifling.

EX-

EXAMPLE II.

*A triangular gable end is raised to the height of 15ft. above the end wall of a house, whose width is 45ft. and the thickness of the wall is 2½ bricks, required the content at the standard measure.*

This figure being a triangle, its contents must be found according to the rules of measuring that kind of figure; then the operation will be as follows :

$$\begin{array}{r}
 45 \text{ width of the house, or base of the triangle.} \\
 15 \text{ height of the gable, or of the triangle.} \\
 \hline
 225 \\
 45 \\
 \hline
 2)675 \\
 \hline
 337.5 \text{ area of the gable, or of the triangle.} \\
 5 \text{ number of half bricks in thickness.} \\
 \hline
 3)1687.5 \\
 \hline
 \text{rds. ft.} \\
 272)562.5(2 \quad 18 \text{ the answer.} \\
 544 \\
 \hline
 18
 \end{array}$$

PROBLEM II.

**To find the quantity of materials for building any wall in brick work.**

Take the dimensions of a building, by measuring half round the outside, and half round the inside, for the whole length of the wall; that being multiplied by the height, will give the area, and proceed as before to find the number of rods or feet reduced to the standard thickness; out of this must be deducted the number of rods or feet, reduced to the same thickness as before, that would be contained in all the vacuities, such as doors, windows, window backs, chimnies,

chimnies, &c. or any other open space in the wall, and the remainder will give the true quantity of rods.

Now the quantity of materials that are generally allowed to a rod of brick work, is 4500 bricks, one hundred and a quarter of lime, and two loads and a half of sand; therefore, if the number of rods or feet are multiplied by 4500, the product will be the number of bricks to a rod of brick work; or if the number of rods, feet, &c. are multiplied by  $1\frac{1}{4}$  hundred weight of lime, will give the quantity of lime; and also if the number of rods are multiplied by  $2\frac{1}{2}$  loads of sand, will give the quantity of sand.

### EXAMPLE I.

*Plate 55, Fig. 1 and 2, let ABCD be the plan or horizontal section of the carcase of any story in a building, the length of the front on the outside is 41ft. 6in.; the length of the end on the inside is 25ft.; the height of the wall or story is 15ft. and 2 bricks thick; in this story are 7 windows, whose vacancies on the outside are 8ft. high, 3ft. 6in. wide, and 1 brick thick; the vacancy in the inside is 4ft. 3in. wide, by 11ft. high, and 1 brick thick; there is also a door 4ft. 6in. wide, by 11ft. high, and 2 chimnies; the breast of each project from the face of the wall 1ft. 6in.; the breadth of the breast is 8ft., the height is 15ft. or the height of the story; the width of the chimney is 3ft. 6in. by 4ft. high; and its depth, from the front to the back, is 3 bricks; required the number of bricks, and the quantity of sand and lime to build the said wall.*

3	6
8	0

---

28	0	area of the vacancy of 1 window on the outside,
7		at 1 brick thick.

---

196	0	area of the vacancies for 7 windows on the outside,
		at 1 brick thick.

Con-

*Continued.*

4	3	
11	0	
<hr/>		
46	9	area of the vacuities of 1 window in the inside, at 1 brick thick.
7		
<hr/>		
327	3	area of the vacuity in the inside for 7 windows, at 1 brick thick.
4	6	
11		
<hr/>		
49	6	area of the vacuity of the door, at 2 bricks thick.
2		
<hr/>		
99	0	area of the vacuity of the door, at 1 brick thick.
3	6	
4	0	
<hr/>		
14	0	area of the vacuity of 1 chimney, within the face of the wall, at 1 brick thick.
2		
<hr/>		
28		area of the vacuities of the 2 chimnies within the face of the wall, at 1 brick thick.
99		
327	3	
196	0	
<hr/>		
650	3	area of all the vacuities, at 1 brick thick, included in the thickness of the wall.
3	6	
4	0	
<hr/>		
14	0	area of one of the vacuities of the chimney, included within the thickness of the breast, at 1 brick thick.
15		
8		
<hr/>		
120		area of one breast, at 2 bricks thick, as if it were solid.
14		deduct the area of the vacuity, at the same thickness.
<hr/>		
106		true area of one breast, at 2 bricks thick.
2		
<hr/>		
12		true area of the two breasts, at 2 bricks thick.

*Con-*

*Continued.*

41	6	length of the front on the outside.
25	0	length of the end in the inside.
<hr/>		
66	6	half the length of the wall.
	2	
<hr/>		
133	0	whole length of the wall.
	15	
<hr/>		
665		
133		
<hr/>		
	1995	area of the wall, at 2 bricks thick.
add	212	the true area of the two breasts, at 2 bricks thick.
<hr/>		
	2207	area of the whole, at 2 bricks thick.
	2	
<hr/>		
	4414	area of the whole, at 1 brick thick.
deduct	650	the areas of the vacuities, at 1 brick thick.
<hr/>		
	3764	true area of the whole, at 1 brick thick.
	2	
<hr/>		
	37528	true area of the whole, at $\frac{1}{2}$ a brick thick.
		rds. ft.
272)2509	4(9	61 reduced to the standard thickness.
	2448	
<hr/>		
	61	

rds. ft.

Then  $9 \ 61 \times 4500 = 41509$  the number of bricks.And  $9 \ 61 \times 1\frac{1}{2} = 11\frac{1}{2}$  hundred weights of lime nearly.Also  $9 \ 61 \times 2\frac{1}{2} = 23$  loads of sand nearly.

EX-

## EXAMPLE II.

*Plate 55, Fig. 3. Let FFGH be a wall of brick work, or the back of a house; to be built over a public road or valley HLG; the under part of the wall is built from the foundation HLG, up to the level at IK, 3 bricks thick, and from IK, to the top EF, parallel to it,  $2\frac{1}{2}$  bricks thick, to the height of 15ft. having 5 windows in it; the vacuities on the outside of each window are 8ft. by 4ft. and half a brick thick; the vacuities in the inside are 8ft. by 4ft. 9in. 2 bricks thick; the recess in the inside for the finishing of the backs in each window, is 1 brick and  $\frac{1}{2}$  thick, the height 2ft. 6in. from the top of the floor to the sill of the window; the width is the width of the vacancy in the inside of the window, that is, 4ft. 9in. There is an arched way underneath for carriages, or the like, to pass through, whose opening is 12ft. and its height from the level of the pavement to the crown or top of the arch is 11ft. and the height, from the pavement to the springing of the arch, is 9ft.; the under wall is divided into an even number of equidistant spaces, whose ordinates are respectively as follows: 6ft. 10ft. 13ft. 14ft. 10ft. 4ft. 6in. and 1ft.; the whole length of the building is 50ft.: required the number of bricks, and the quantity of sand and lime to build the said wall.*

## EXPLANATION.

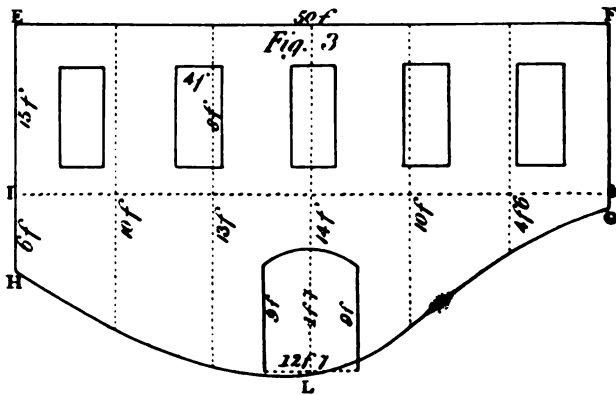
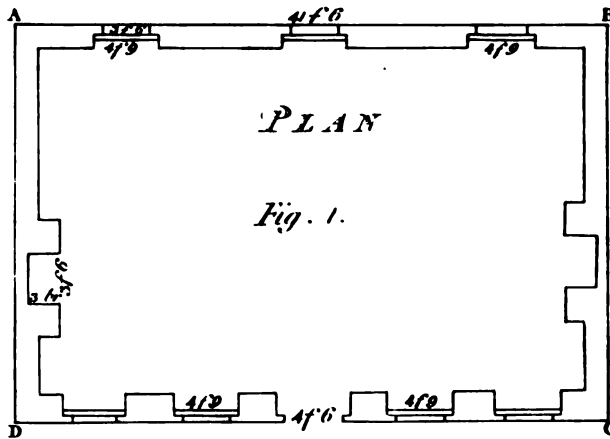
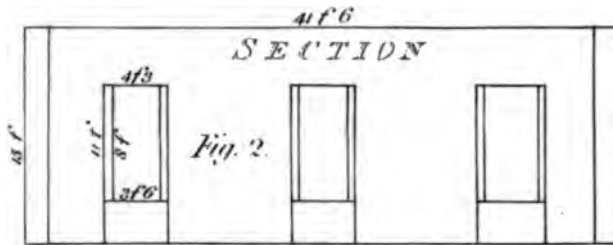
The under part of the building being an irregular figure, it is measured according to Method II. p. 213; but instead of multiplying by the common distance, it is multiplied by the length of the building, and the product divided by the number of spaces, which amounts to the same thing: the upper part is found, as in the foregoing examples. The rule for measuring the arched way, or any other arched window or doors whatever, may be as follows: to twice the height of the window or door, taken from the crown of the

arch to the sill or bottom, add the height from the bottom or sill, to the springing of the arch; multiply the sum by the width of the window or door, and one third of the product will be the area near enough for practice: but if greater accuracy is required, add the cube of the altitude, divided by twice the width of the arch, door, or window, &c. to the area found as before, and it will give the content exceedingly near the truth. The windows are measured by finding the area of the outside vacuity, and the area of the inside vacuity; and as the back of the windows, from the sill of the floor, is generally built half a brick less from the inside, than the upper vacuity on the same side, in order to support the bottom of the sash frame, the area of that part or vacuity is also to be found; multiply each respective area by the number of half bricks it is in thickness; then deduct all the vacuities at half a brick thick from the area of the whole found as if it were solid, at half a brick thick, as before; the remainder being divided by 3, will reduce it to the standard thickness of 1 brick and  $\frac{1}{2}$ .

10	13
14	10
4 6	—
—	23
28 6	2
4	—
—	46
114 0	four times the sum of the even ordinates.
add 46	twice the sum of the odd ordinates.
—	
3)160	
—	
53 4	
50	
—	
6)2666 8	
—	
444 5	area of the under part of the wall, 3 bricks thick.
6	number of half bricks.
—	
2666 6	area of the under part of the wall, $\frac{1}{2}$ a brick thick.

Con.

# ARTIFICERS WORK





*Continued.*

15 height of the upper part of the wall.  
50 length.

---

750 area of the upper part of the wall,  $2\frac{1}{2}$  bricks thick.  
5 number of half bricks.

---

3750 area of ditto,  $\frac{1}{2}$  a brick thick.

8  
4

---

32 area of the vacuity on the outside,  $\frac{1}{2}$  a brick thick.  
5 number of windows.

---

160 area of the vacuities on the outside of five windows,  
 $\frac{1}{2}$  a brick thick.

8 4  
4 9

---

33 4  
6 3

---

39 7 area of the vacuity of the inside for 1 window, 2  
5 number of windows. [bricks thick.

---

197 11 area of the vacuities for five windows in the inside.  
4 half bricks thick.

---

791 8 area of the vacuities on the inside,  $\frac{1}{2}$  a brick thick.

4 9  
2 6

---

9 6  
2 4 6

---

11 10 6 area of the vacuity of the recess under each  
5 window, 1 brick and a  $\frac{1}{2}$  thick.

---

59 4 6 area of the vacuities of the 5 window backs, 1  
3 number of half bricks. [brick and a half thick.

---

178 1 6 area of the vacuities of the 5 window backs,  $\frac{1}{2}$  a  
brick thick.

Continued.

11 height of the arch-way, from the pavement to the  
2 crown.

22

add 9 height from the pavement to the springing of the arch.

31

10 width of the arch-way.

3)310

103 4 area of the vacuity of the arch-way, 3 bricks thick  
6 number of  $\frac{1}{2}$  bricks thick.

620 area of the vacuity of the arch-way,  $\frac{1}{2}$  a brick thick.

160

791 8

178 1

690

1749 9 areas of all the vacuities,  $\frac{1}{2}$  a brick thick.

2666 6

3750

6416 6 area of the whole,  $\frac{1}{2}$  a brick thick, as if solid.

deduct 1749 9

3)4666 9 true area of the whole,  $\frac{1}{2}$  a brick thick.  
rds. ft.

272)1555 7 (5 195 reduced to the standard thickness.

1360

195

Then  $5 \ 195 \times 4500 = 25726$  number of bricks, nearly.

$5 \ 195 \times 1\frac{1}{2} = 7$  hundred weight of lime, nearly.

$5 \ 195 \times 2\frac{1}{2} = 14$  loads of sand, nearly.

I shall,

I shall, in the following Examples, shew the manner of finding the areas of windows, angle chimnies, and the different stories of a building.

### PROBLEM III.

To measure the vacuity of a window.

Find the area of the outside of the window, and multiply that by the number of half bricks thick, from the face of the sash frame on the outside, to the face of the wall of the same side, to the area so found, at half a brick thick; add the area of the inside vacuity multiplied by the number of half bricks thick, from the face of the sash frame on the outside, to the face of the brick work within the building: also add the area of the vacuity of the recess, the height being taken from the bottom of the sash frame to the floor, and its width the same as the inside vacuity above; multiply this also by the number of half bricks thick, then the sum of these will be the whole vacuity, or void space in the whole window, at half a brick thick; and if required to be reduced to the standard, divide the area so found by 3, and the area of the contents will be reduced to 1 brick and a  $\frac{1}{2}$  thick.

EX.

## EXAMPLE I.

Let Fig. 3, Plate 56, be the plan or horizontal section of a window. Fig. 1, the elevation as would appear within the building. Fig. 2, a vertical section through the middle of the elevation of the window; the height of the outside vacuity is 8ft. and its breadth 4ft. and half a brick thick; the height of the inside vacuity is 8ft. and its breadth 4ft. 9in. and 2 bricks thick, as appears by the plan and section; the recess is 2ft. 9in. high, 4ft. 9in. wide, and half a brick thick, which is also marked upon the plan and section; required the area of the whole vacuity at half a brick thick.

8 height of the outside vacuity.

4 width of the outside vacuity.

32 area of the outside vacuity,  $\frac{1}{2}$  a brick thick.

4 9 width of the inside vacuity.

8 height of ditto.

---

38

4 number of half bricks.

---

152 area of the inside vacuity,  $\frac{1}{2}$  a brick thick.

4 9 width of the inside vacuity.

2 9 height of the recess from the floor to the side of the  
sash,

---

9 6

3 6 9

---

13 0 9 area at 1 brick and a  $\frac{1}{2}$  thick.

3

---

39 2 3 area of the recess,  $\frac{1}{2}$  a brick thick.

32

---

152

---

223 2 3 area of the whole vacuity,  $\frac{1}{2}$  a brick thick.

PROBLEM

### PROBLEM IV.

To measure any angle chimney, standing equally distant each way from the angle of the room.

Plate 58, Fig. 4. Multiply the breadth AB by the height of the story, and the product by the number of half bricks contained in the half breadth AB, and you will have the solidity at half a brick thick, by deducting the vacuity or opening of the chimney.

### PROBLEM V.

To measure an angle chimney, when the plane of its breast intersects the two sides of the room unequally distant from the angle.

Fig. 5. From the points A and B, where the plane of the breast intersects the sides of the room, draw two lines, AE, EB, parallel to the two sides of the room; then multiply either of the lines AE or EB, suppose EB, by the height of the room, and multiply that product by the number of half bricks contained in the other line AE, and deduct the vacuity as before, and the remainder will be the content, at half a brick thick.

### PROBLEM VI.

To measure an angle chimney, when the plane of the breast projects out from each wall, and unequally distant from the angle of the room.

Draw the two lines GF and FH parallel to the two sides of the room, as before; then multiply the breadth FH by  
the

the height of the story, and the product contained in the half of the other side FG; from this deduct FD, multiplied by the height of the story, and by the number of half bricks contained in the half of FC, and also the vacuity of the chimney.

## PROBLEM VII.

**To measure the whole carcase of a building, consisting of different stories.**

1. Fig. 6. First begin with the foundation, which in general consists of three or more courses of brick, projecting equally each way over one another, according to the weight of the building above, and the solidity of the ground underneath. The method of measuring this may be as follows:

2. Multiply the whole length of the foundation by the height of the courses, and the product by the number of half bricks contained in the half sum of the breadths of the top and bottom courses, and the last product will give the solidity.

3. Proceed and measure every story separately, as if solid, and reduce each solidity to the thickness of half a brick.

4. Add the solidities so found in all the different stories together.

5. Find the solidities of all the vacuities in the building, except the funnels of chimnies, at half a brick thick.

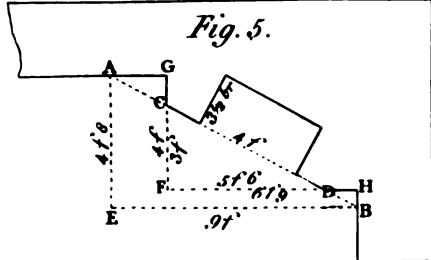
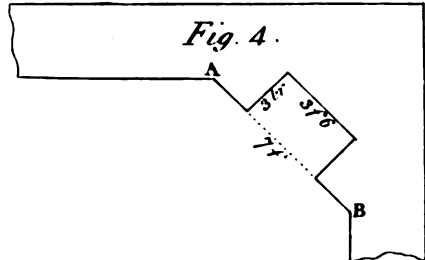
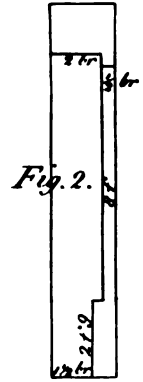
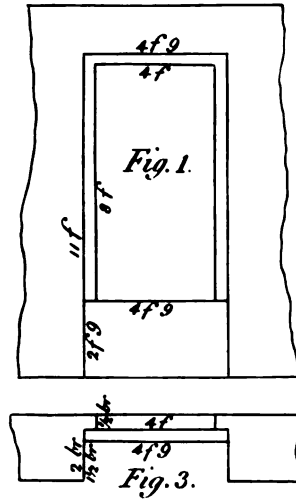
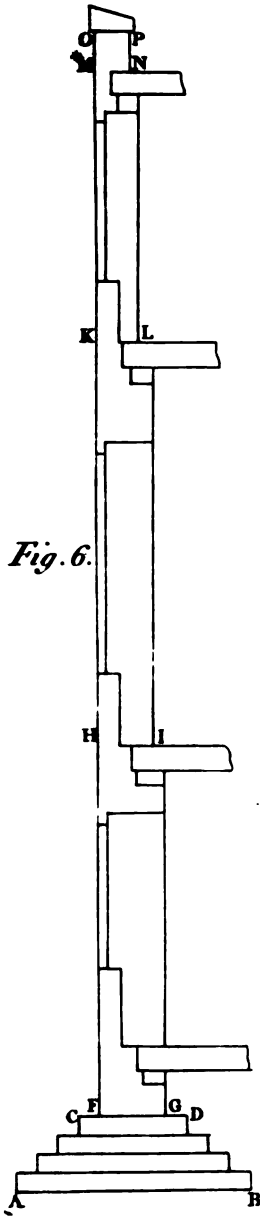
6. Deduct the solidities that would be contained in all the vacuities, from the solidity of the whole, and the remainder will be the true solidity at half a brick thick.

7. Divide the remainder by 3, and it will be reduced to a brick and a half.

8. Divide the solidity found, at a brick and a half thick, by 272, and the quotient will be the number of rods in the whole building.

CAR-

# ARTIFICERS WORK





# CARPENTERS' & JOINERS' WORK.

## DEFINITION.

By Carpenters work, is meant the measuring of groined centres, floors, partitions, roofs, &c.

## PROBLEM I.

To measure the centering of a cylindrical vault.

Multiply the length of the vault in feet, by the circumference of the arch, for the breadth, and divide the product by 100; if it is greater than the same, the quotient will give the number of squares and feet.

## EXAMPLE I.

How many squares of centering are there in a vault, whose length is 18ft. 6in. and the circumference 31ft. 6in.?

By duodecimals.

By vulgar fractions.

By decimals.

ft. in.  
 31 6  
 18 6  
 ———  
 248  
 310  
 9  
 15 6  
 0 3  
 ———  
 5,82 9  
 or 5 squares 82 feet.

31½  
 18½  
 ———  
 157½  
 9  
 248  
 31  
 ———  
 5,82½

31.5  
 18.5  
 ———  
 1575  
 2520  
 315  
 ———  
 5,8275

## PROBLEM II.

To measure naked floors, whether for materials or workmanship.

## EXPLANATION

If there are any number of pieces of timber of the same scantling and length, find the solidity of one of them, and that solidity multiplied by the number of them, will give the solidity of the whole.

## EXAMPLE

If there are any number of pieces of the same scantling, but of different lengths, add all the different lengths together, and multiply the sum by the area of the end of one of the pieces, and the product will give the solidity of the whole.

Example: If there be 10 pieces of timber, each 12 feet long, and 12 inches square at the end, the solidity will be 1728 cubic feet.

If there are any number of pieces of different scantlings, but of the same length, find the areas of the ends of all the pieces, and the sum of these areas being multiplied by the common length, will give the solidity of the whole number.

If there should be any number of pieces of one scantling, equal among themselves, and any other number of pieces of another scantling, equal among themselves, all of the same length; multiply the area of the ends of each, by the number of them that are of the same scantling, add these products together, and their sum being multiplied by the common length, will give the solidity.

But if the lengths vary, as well as the scantlings, find the solidity of each piece separately, and the sum will give the solidity of the whole.

Note.

*Note.* Wherever a tenant is made, the length of the piece must be taken from the ends of the tenants, and not from the shoulders.

If the floors are fixed in the building, the distance the timber goes into the wall, which is about  $\frac{1}{2}$  of the thickness of the wall, must be added to the length of their respective pieces that are clear of the walls.

Before I proceed to give an example in measuring a floor, it will be proper to explain the several pieces of timber that constitute the same.

#### EXPLANATION OF THE TIMBER IN A FLOOR.

Let Plate 57, Fig. 1, be the plan of a naked floor, Fig. 2 and 3 are sections each way; the girder is marked A, and the section of its end is marked a in Fig. 3; the binding joists are marked B, B, B, B, &c. the sides are marked b, b, in Fig. 3; the ends are marked b, b, b, b, in Fig. 2; the bridging joists are marked C, C, C, &c. in the plan at Fig. 1, the ends are marked c, c, c, &c. at Fig. 3, and the side is marked c at Fig. 2; the ceiling joists do not appear on the plan at Fig. 1, because of the bridging joists appearing before them; the ends are marked e, e, e, at Fig. 3, and the sides are marked e, e, e, &c. at Fig. 2.

## EXAMPLE.

Let Fig. 1, Plate 57, be the plan of a floor, as before; suppose the girder, marked A, to be 1ft. broad, 1ft. 2in. deep, and 20ft. long; there is 8 bridging joists marked C, C, C, &c. whose scantlings are 3in. by 6½in. and 20ft. long, that is of the same length with the girder; there is also 8 binding joists, whose lengths are 9ft. and their scantlings are 8½in. by 4in.; the ceiling joists are 24 in number, each 6ft. long, 4in. by 2½in.; required the solidity of the whole, either for materials or workmanship.

1 2  
1 0

---

1 2 area of the end of the girder.

i. ii  
6 6  
3

---

1 7 6iii area of the end of a bridging joist.  
8 number of bridging joists.

---

1 1 Oii Oiii  
add 1 2 the area of the end of the girder. [bridging joists.  
2f. 3in sum of the areas of the ends of the girder and  
20 0 common length.

---

45f. 0 the solidity of the girder and bridging joists.

i. ii.  
8 6 depth.  
4 thick.

---

2i 10ii Oiii  
9

---

2f. 1 6 solidity of a binding joist.  
8

---

17f. 0 0 solidity of all the binding joists.

Con-

*Continued.*

2 6ii  
4i

---

10ii 0iii area of the end of a ceiling joist.  
6

---

5i 0 solidity of a ceiling joist.  
24 number of ceiling joists.

---

10 0 solidity of all the ceiling joists.  
17 0  
45 0

---

72 0 sum of all the solidities in the whole floor.

### PROBLEM III.

To measure roofing or partitions, either for materials or workmanship.

All timbers in a roof or partition, are measured in the same manner as floors, excepting king-posts, queen-posts, &c. when there is a necessity of cutting out parallel pieces of wood from their sides, in order that the ends of the braces that come against them, may have, what is called by workmen, a square butment. To measure the workmanship of such pieces, or posts, take their breadth and depth at the widest part, and that multiplied by the length, will be the solidity for workmanship. To find the quantity of materials, if the pieces sawn out are  $2\frac{1}{2}$  inches thick, or more, they are esteemed pieces of timber fit for use; when more than 2 feet long, their lengths should not be esteemed so long by 5 or 6 inches, because the saw cannot enter the wood with much less waste, and consequently the pieces must be deducted

deducted from the whole solidity, and the remainder will give the quantity of materials: but if the pieces cut out are less than  $2\frac{1}{2}$  inches, then the whole post is measured as solid for the materials, because the pieces cut out are but little use.

## EXAMPLE.

Plate 57, Fig. 4, let the tie-beam D be 36ft. long, 9in. wide, by 1ft. 2in. deep; the king-post, marked A, is 11ft. 6in. high, 1ft. broad at the bottom, by 5in. thick: out of this are sawn two pieces from the sides, 3in. thick, and 7ft. long; the braces, marked B, B, are 7ft. 6in. long, 5in. by 5in.; the rafters, marked D, D, are 19ft. long, 10in. by 5in. each; the struts, marked C, C, are 3ft. 6in. long, and 4in. by 5in; required the measurement for workmanship, and also for materials.

$$\begin{array}{r} 1 \quad 2i \\ 9i \\ \hline 10 \quad 6 \\ 3 \quad 0 \\ \hline \end{array}$$

31 6 solidity of the tie beam.

$$\begin{array}{r} 5i \\ 5i \\ \hline 2 \quad 1ii \\ 15 \quad \text{lengths of the 2 braces added together.} \\ \hline \end{array}$$

2f. 7 3

$$\begin{array}{r} 5i \\ 10i \\ \hline 4 \quad 2i \\ 3 \quad 2i \\ \hline 8 \quad 4ii \\ 12 \quad 6 \\ \hline \end{array}$$

19f. 2 4iii solidity of both rafters.

Con-

# ARTIFICERS WORK

Fig. 2.



Fig. 1.

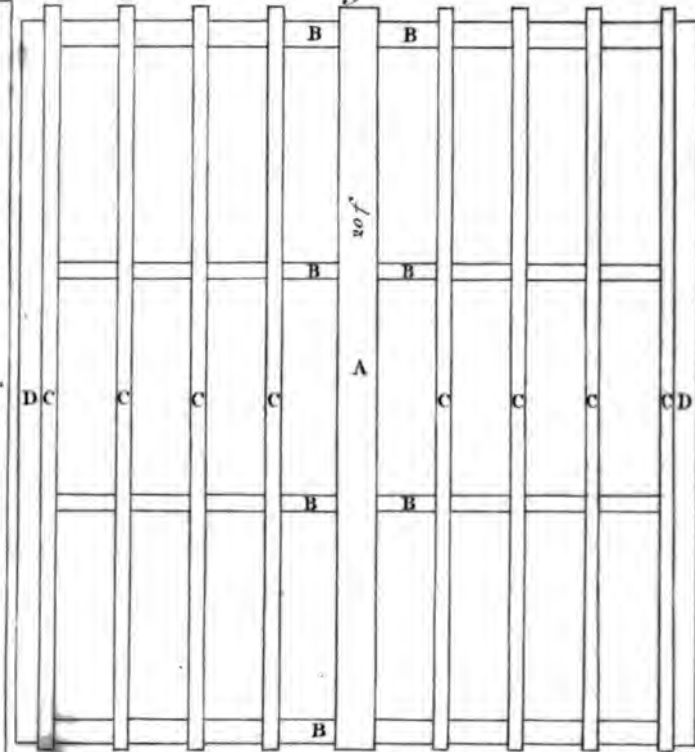


Fig. 3.

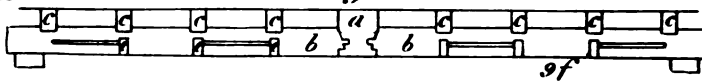
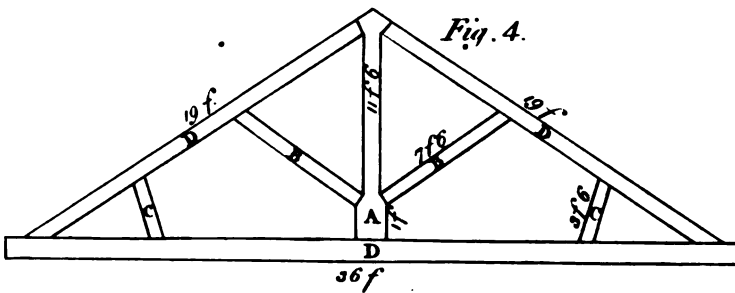


Fig. 4.





*Continued.*

	ft.	
	1	
	5i	
	—	
	5i	
11	6i	
<hr/>		
4	9	6 solidity of the king-post, as if solid.

	3i	
	5i	
<hr/>		
1i	3ii	
76		
<hr/>		
8i	9ii	solidity of the two pieces cut from the sides.

	4i	
	5i	
<hr/>		
1i	8ii	
7f.		
<hr/>		
11i	8ii	solidity of the struts,

*Consequently,*

f.	i.	ii.	
31	6	10	tie beam.
2	7	3	braces.
13	2	4	rafters.
4	9	6	hinge post.
11		8	solidity of the struts:
<hr/>			
53	0	9	solidity of the roof for workmanship.
	8	9	
<hr/>			
52	4	0	solidity for materials.

I shall here finish Carpenters and Joiners' work, by shewing several other customs in measuring sundry articles not spoken of in this treatise.

1. In

1. In boarded flooring, you must take your dimensions to the very extreme parts, and from thence compute the squares, out of which you must make deductions for stair-cases, chimnies, &c.

2. Weather-boarding is done by the yard square, and sometimes by the square, containing 100 superficial feet.

3. Beadled partitions are measured by the square, out of which you must deduct the doors and windows contained therein, except they are agreed to be included.

4. Windows are generally made and valued by the foot, superficial measure, and sometimes by the window. When they are measured, the dimensions must be taken in feet and inches, from the under side of the sill to the upper side of the top rail, for the height; and for the breadth, from outside to outside of the jambs; and the product of these is the superficial content.

5. Stair-cases are measured by the foot superficial, and the dimensions are taken with a string, girt over the riser and tread; and that length or girt, multiplied by the length of the step, will produce the superficial content.

6. The rail is taken at so much per foot run, according to the diameter of the well-hole; which price will be greater when the well-hole is less. Architrave string boards, by the foot superficial. Brackets and strings, at per piece, according to the workmanship of them.

7. Door-cases are measured by the foot superficial, and the dimensions must be taken with a string, girt round the architrave and inside of the jambs, for the breadth; and for the length, add the length of the two jambs to the length of the cap-piece, taking the breadth  
of

of the opening for the length, and the product is the superficial content.

8. Frame doors are measured by the foot, or sometimes by the yard square, containing nine square feet, and are valued at so much per foot, according to the workmanship.

9. *Note.* The same is to be said in regard to the measuring and valuing window-shutters, as of doors.

10. Modillion cornishes, coves, &c. are generally measured and valued by the foot superficial. Their dimensions, in respect to the breadth, are taken with a string, girt into the mouldings, and those dimensions multiplied by the length, is the superficial content.

11. Wainscoting is a work generally done by Joiners, and is measured by the yard square, and their dimensions are taken by feet and inches. Thus they girt down every moulding with a string, contained between the floor and ceiling, for the height, and the circumference of the room for the length, deducting the doors, windows, and chimney. The seats of windows, if any, cheeks, soffits, linings, &c. are all to be taken by themselves.

12. Frontispieces are measured and valued by the foot superficial, and every part measured separately, viz, the architrave, frieze, and cornish, each of them by themselves; and lastly, add all the several measurements together, and the product is the content of the whole.

*Note.* In taking the dimensions, you must girt the mouldings with a string.

## MASONS' WORK.

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Masons' work is measured by foot measure, either lineal, square, or cubical. The principal thing to be observed here is, that they girt all their mouldings as Joiners do, and take their dimensions in feet, inches, and parts.

The solids are blocks of stone, marble, or any kind of stone, columns, cornishes, &c. The superficies are pavements, slabs, chimney-pieces, and the like. It is to be observed, that Masons first measure the cube of the stone, and then the superficial plain work; also superficial moulded work, if any, as follows:

They account all such stones as are above 2 inches thick, at so much per foot solid measure; and for the workmanship they measure the superficies of the stone; but then they measure no more of the stone than what appears without the wall

---

## PLASTERERS' WORK.

---

Rough-casting, plastering ceilings, &c. are done by the yard square, and the dimensions taken in feet and inches.

The

The principal things to be observed in measuring of which, are as follows :

1. To make deductions for chimnies, windows, and doors.
2. To make deductions for rendering upon brick-work, for doors and windows.
3. If the workmen find materials for rendering between quarters, you must deduct one fifth for quarters; but if workmanship only is found, you must measure the whole as whole work, for the workman could have performed the whole much sooner, if there had been no quarters.
4. That such summers and girders as lie below a ceiling be deducted, if the workman find materials; otherwise not.
5. All mouldings in plaster work, are done by the foot superficial, as Joiners do, by girting over the mouldings with a line.

---

## GLAZIERS' WORK.

---

Glaziers' work is measured by the superficial foot, and the dimensions are taken in feet, inches, and parts, or by feet, and the hundred parts of a foot, as their rules are generally divided in decimal parts; and consequently their dimensions are squared according to the rules of decimals.

L 1 2

To

To measure circular or oval windows, take the same length and breadth as their diameters, as if they had been square windows; because in cutting out the squares of glass, there is great waste, and more time expended, than if they had been square windows.

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## PAINTERS' WORK.

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Painters' work is measured the same as Joiners' work, by the yard square, (See p. 257.) and they also measure all edges, &c. where the brush goes.

1. Sash-frames, sash-lights, window-lights, and casements, are done at per piece.

2. Modillion, and other outside cornishes, at per foot, running.

---

---

## GROINS.

---

### DEFINITION.

Groins are the intersection of two segments of a cylinder, each of the same altitude meeting on their diagonal sections; the base of each segment is supposed to be in the same plane, and parallel to the axes of each cylinder.

Or

Or groins may be the intersection of a cylindrical segment with a cylindroidal one, each of the same altitude, and their bases in the same plane meeting on their diagonal sections.

## PROBLEM I.

To find the superficies of a groin.

### CASE I.

When the sides of the groin are semicircles, to the area of the base add  $\frac{1}{4}$ th part of itself, and the sum will give the superficies required.

### EXAMPLE I.

*What is the curve superficies of a circular groin, each side of the square base being 12ft.?*

$$\begin{array}{r}
 12 \\
 12 \\
 \hline
 144 \text{ area of the base.} \\
 20.57 \\
 \hline
 164.57 \text{ area of the groin.}
 \end{array}$$

### CASE II.

When the groin stands upon a rectangular plan, the sides being either segments of circles, or segments of an ellipsis.

The area of each two opposite parts of the surfaces of the cylinders, or cylindroids, may be computed in the following manner.

Let

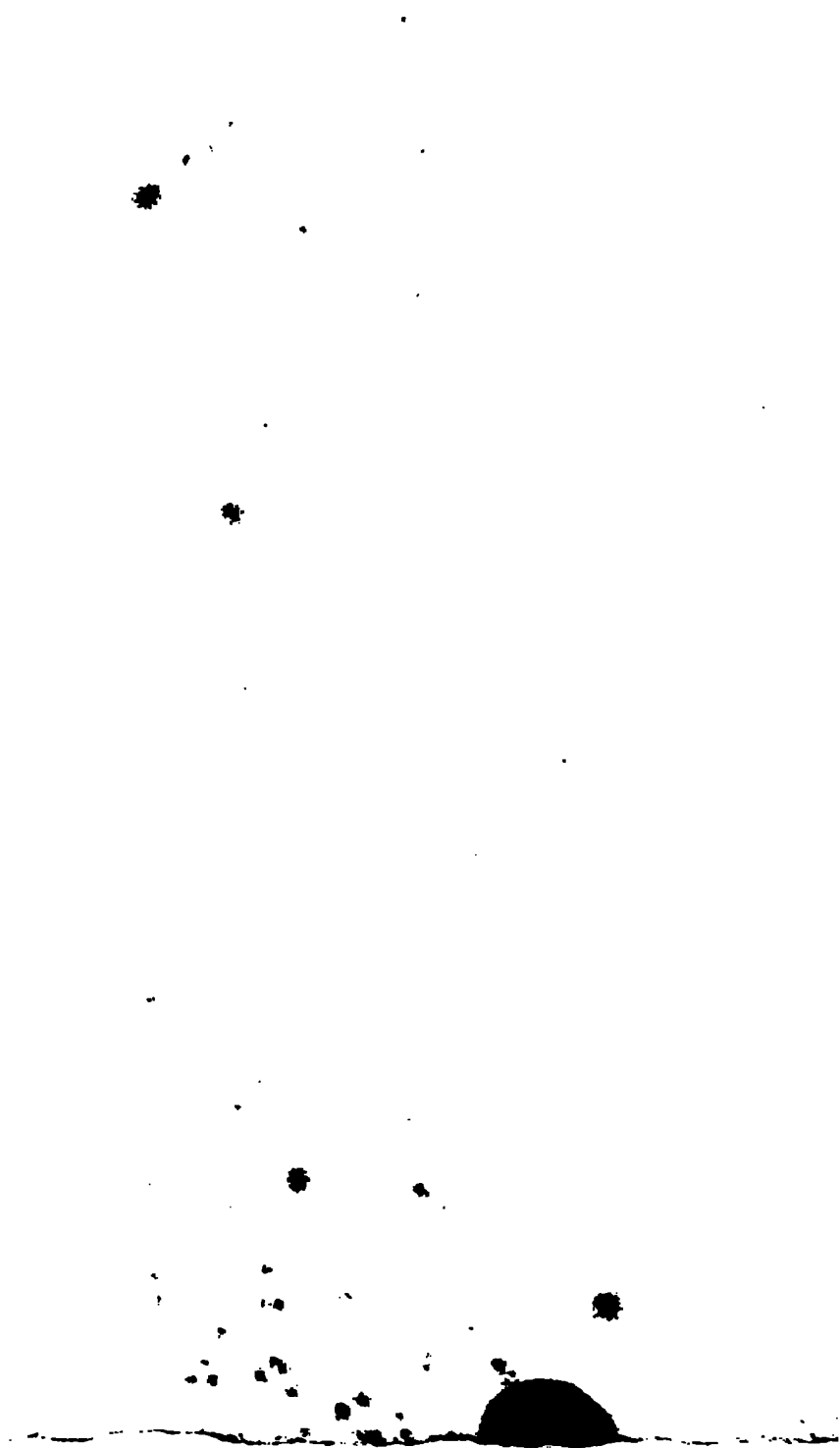
Plate 58, Fig. 1 and 2. Let ABCD be the plan of a groin; AC and BD are the intersection of the planes of the diagonals; MNQO is one of the cylindrical, or cylindroidal surfaces stretched out on a plane; HVI is one of the side arches. Then to find the area of any two opposite quarters of the cylindrical, or cylindroidal surfaces, to the arch line HVI, standing over BC on the plan; that is, NM, when stretched out on the plane, add four times the arch standing over FG on the plan taken in the middle, between the end BC, and the vertex at E, that is, QR, when stretched out on the plane; multiply one third of the sum by ES, or PO, which is equal to it; and the product will be double the area of the two opposite cylindrical or cylindroidal surfaces, standing over AED and BEC.

## EXAMPLE I.

Let ABCD be the plan of a groin, the sides AB and BC are each equal to 8ft.; let MN, the length of the arch HVI, standing over BC on the plan, be 10ft. and PO, equal to ES, be 4ft.; that is, the distance measured along from the crown of the arch, taken from the vertex at E, to either of the ends at S, and QR the length of the arch over FG, be 4ft.; required the superficies of the groin.

$$\begin{array}{r}
 4 \\
 4 \\
 \hline
 16 \text{ four times QR.} \\
 10 \text{ the end.} \\
 \hline
 3)26 \\
 \hline
 8 \quad 8i \\
 4 \\
 \hline
 34 \quad 8 \text{ area of the two opposite parts AED, \& BEC.} \\
 2 \\
 \hline
 69 \quad 4 \text{ area of the whole groin, standing over} \\
 \text{ABCD, on the plan.}
 \end{array}$$

EX-



*Shortly will be published in 1 Vol. Quarto,*

BY THE SAME AUTHOR,

AN



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Explanations of the Terms used by Architects and Workmen, in Building; with the Theory and Practice of the various Branches in Architecture, and their dependence on each other, so as to form one complete system of the Science and Art of Building. Likewise, a description of the Materials, shewing their qualities, proportions, and quantities; also the best method of preparing them, with specifications or particulars of work, according to their various classes. In which will be introduced, all the useful Geometrical Diagrams in Carpentry, Masonry, &c. many of which were never before published.

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